INSTITUTION OF
POST OFFICE ELECTRICAL ENGINEERS.

THE LOADING OF TELEPHONE CABLE CIRCUITS.

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A PAPER

read before the Institution of Post Office Electrical Engineers

ON JANUARY 11th and FEBRUARY 8th, 1909.

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INTRODUCTION.

IN any telephone circuit there are at least four factors which exert a marked influence upon the transmission of the electric impulses, waves or rapidly alternating currents, which are convertible into speech by means of a telephone receiver. These factors are :—

(*a*) Resistance.
(*b*) Inductance.
(*c*) Electrostatic capacity.
(*d*) Leakance (reciprocal of insulation resistance).

There is no definite relationship between these quantities in the sense that if three be known the fourth can be determined ; consequently, the investigation of their joint influence upon the transmission of speech is a matter of some complexity. It is not, however, so complex but that anyone acquainted with the ordinary laws connecting the well-known electrical quantities may follow the consideration if a little mathematical knowledge be possessed.

Four quantities have been named above ; but there are others not so obviously included. When any current of varying strength—particularly one of high frequency such as a telephonic current—flows through a circuit, eddy currents are produced in any mass of conducting material near the main conductor, and such currents react upon those producing them so as to retard the rise and fall in strength. Again, if there be iron near the main conductor, the iron will be reversely magnetised with each alternation of the current, and for every cycle through which the magnetisation is carried there will be a definite hysteresis loss in the iron.

So far as the main, or telephone, circuit is concerned, there is a certain loss of energy due to eddy currents and the hysteresis in the iron. It should be noted that these losses are *absolute* and not comparable with inductance and capacity effects in which energy is stored, not lost. The energy lost in any circuit is proportional to C^2R, where C is the current strength and R the resistance. If eddy currents and hysteresis losses produce the same effect as a resistance in limiting the current strength, they may be represented by a resistance, and, clearly, such a so-called resistance is an addition to the ordinary resistance of the circuit as measured under steady current conditions. The term *effective resistance* is used to denote ohmic resistance (or that offered to steady currents) plus that due to the effect of eddy currents and hysteresis.

In any ordinary telephone circuit consisting of overhead, underground or submarine wires revolved, without special treatment, the effective resistance is not very much greater than the ohmic resistance ; indeed, the difference is so small as to be practically negligible. *Do not, however, imagine that this is the case where inductance or loading coils are placed in a circuit, or where inductance is increased by coating the conductor with magnetic material, such as a layer (or layers) of iron wire.* As will be mentioned later, the effective resistance in these cases is vastly different from the ohmic ; either owing to the presence of the mass of iron used in the loading coils and the copper of which the coils are formed, or to the mass of iron distributed uniformly over the conductor as the case may be.

The subject of effective resistance has been emphasised early, because it is a matter of the utmost importance, and is unfortunately overlooked by many observers. Methods of increasing the inductance of a circuit are not difficult to find ; the real difficulty is to devise a method of introducing inductance without largely increasing the effective resistance.

In circuits free from iron, the inductance may be regarded as constant for all currents and frequencies ; but if iron be present, as in the case of circuits loaded with iron core coils, this is not absolutely true. By properly selecting the quality and arranging the quantity of iron or other magnetic material used, a very close approximation to uniformity of inductance values for all important telephonic frequencies may, however, be obtained. In what follows, it will be understood that the inductance considered is that measured at the frequency assumed in the particular cases ; but it may at once be stated that for all frequencies between 500 and 2,000 per second, the loading coils (with or without iron) used in practice are constant in inductance for telephonic currents.

PART I.

Any treatise on algebra dealing with exponential and logarithmic series will show that, if—

$$e = 1 + 1 + \frac{1}{\underline{|2}} + \frac{1}{\underline{|3}} + \ldots \&c.,$$

then $\text{M}\, \epsilon^{ax} = \text{M} \left(1 + ax + \frac{a^2\, x^2}{\underline{|2}} + \frac{a^3\, x^3}{\underline{|3}} + \ldots \&c.\right)$

where M is a constant.

If this expression be differentiated with respect to x, then

$$\frac{d\,(\text{M}\, \epsilon^{ax})}{dx} = \text{M} \left(0 + a + a^2\, x + \frac{a^3\, x^2}{\underline{|2}} + \frac{a^4\, x^3}{\underline{|3}} + \ldots \&c.\right),$$

$$= \text{M}\, a \left(1 + ax + \frac{a^2\, x^2}{\underline{|2}} + \frac{a^3\, x^3}{\underline{|3}} + \ldots \&c.\right),$$

$$= \text{M}\, a\epsilon^{ax}.$$

Put $y = \text{M}e^{\,ax}$, and the following relation is obtained—

$$\frac{dy}{dx} = ay.$$

From this it is argued that when the rate of increase of any quantity, y, with respect to another, x, is proportional to itself, it obeys the relation—

$$y = \text{M}\, e^{ax}.$$

If a further step be taken and the expression for $\text{M}\, e^{ax}$ be differentiated twice, then—

$$\frac{d^2 y}{dx^2} = \frac{d\,(\text{M}\, a\, \epsilon^{ax})}{dx} = \text{M}\, a^2\, e^{ax},$$

or, since $y = \text{M} e^{ax}$,

$$\frac{d^2 y}{dx^2} = a^2 y.$$

This is all very straightforward when one meets with $y = \text{M} \epsilon^{ax}$, but when one begins with an equation such as—

$$\frac{d^2 y}{dx^2} = a^2 y,$$

it does not follow that—

$$y = \text{M} \epsilon^{ax} \textit{ simply}.$$

Let $y = \text{M} \epsilon^{ax} + \text{N} e^{-ax}$,

then $\frac{dy}{dx} = \text{M} a e^{ax} - \text{N} a e^{-ax}$,

and

$$\begin{aligned} \frac{d^2y}{dx^2} &= \text{M} a^2 \epsilon^{ax} + \text{N} a^2 e^{-ax}, \\ &= a^2 (\text{M} \epsilon^{ax} + \text{N} e^{-ax}), \\ &= a^2 y. \end{aligned}$$

By trial, therefore, it is seen that—

$$y = \text{M} \epsilon^{ax} + \text{N} \epsilon^{-ax}$$

is a solution of the equation—

$$\frac{d^2y}{dx^2} = a^2 y.$$

(See Boole's "Differential Equations," Second Edition, Chapter IX, page 194.)

As a matter of fact, one may put y equal to as many terms in e as one pleases, so long as the powers to which e is raised are kept equal to ax and $-ax$ respectively. In the above expression M stands for the algebraic sum of all the coefficients of terms in ϵ^{ax} and N represents that of all the coefficients of terms in ϵ^{-ax}.

Proceeding now to the consideration of a telephone circuit, let it first be understood that every sound used in normal speech is composed of a number of superposed sound waves. Suppose three of these waves to be as shown by a, b and c in fig. 1, the resulting wave would be of the form shown by d. It is unnecessary here to enter into detail as to the manner of converting sound waves in air into currents of electricity, of varying strength, in an electric circuit, but it is essential to bear in mind that if perfect telephonic transmission is to be obtained, the circuit must be capable of transmitting currents, the variation in the strengths of which, if plotted, would show the same resultant wave form as the speech wave itself—such as *d*, fig. 1. The variation of

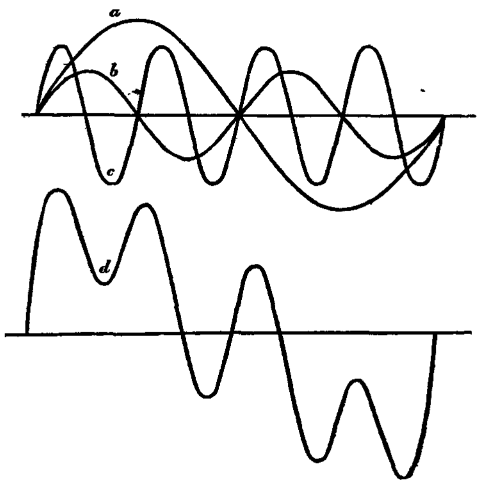


FIG. 1.

telephonic voltages and current strengths clearly may be assumed to obey the general laws for periodic E.M.F.'s and currents of high frequency.

The following symbols will be used hereafter, others being employed and defined as required :—

R = effective resistance in ohms per mile of loop.

L = effective inductance in henrys per mile of loop.

K = electrostatic capacity in farads per mile wire to wire.

S = leakance in mhos per mile wire to wire. (Or reciprocal of the insulation resistance in ohms per mile wire to wire.)

p = 2π times the periodicity per second.

In fig. 2, let A B be a looped circuit ; at A let there be a means of impressing an alternating E.M.F. Take any extremely small length of the loop—say, at P—and call it Δx. Suppose that, at the instant considered, the upper wire as viewed in the figure, is at a positive, and the lower at a negative potential; and, that the dotted lines represent the way in which the potentials change along the wires of the loop in the neighbourhood of P. Next, for the sake of convenience, imagine the short length Δx to be re-drawn on a very enlarged scale as in figs. 3 and 4. Consider, first, fig. 3. If C be the instantaneous current strength in the length of loop Δx, then the fall of potential difference

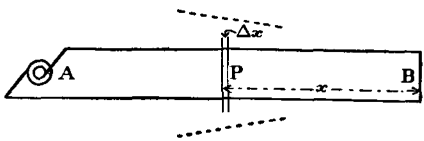


FIG. 2.

between the wires in the direction A to B (see fig. 2), or the rise from B to A, due to the joint effects of the resistance $R \Delta x$ and the inductance $L \Delta x$, will be $C R \Delta x + L \Delta x \frac{dC}{dt}$, so that, if V be the difference of potential at the right end, that at the left will be $V + \left(CR + L \frac{dC}{dt} \right) \Delta x$. As Δx is extremely small, no grave error will arise if it be assumed that the instantaneous difference of potential between the upper and lower wires of the short length is V, and that ΔV represents the small change of potential difference between the ends of Δx. In which case, and considering B to A—

$$\Delta V = \left(CR + L \frac{dC}{dt} \right) \Delta x,$$

$$\frac{\Delta V}{\Delta x} = CR + L \frac{dC}{dt} \text{ nearly,}$$

and if Δx be made less than any assignable magnitude then, absolutely—

$$\frac{dV}{dx} = CR + L \frac{dC}{dt}. \quad \ldots\ldots\ldots (1)$$

Now, consider fig. 4; where the length Δx is the same as in the preceding case. The leakance between the two wires will be $S\Delta x$, and the electrostatic capacity

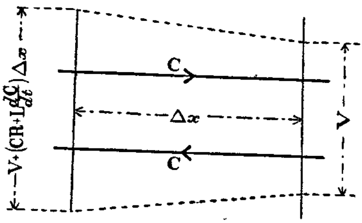


FIG. 3.

$K\Delta x$. If the instantaneous difference of potential between the wires of the short length of loop be V, then the current leaking across the dielectric from wire to wire will be $V\,S\,\Delta x$. In addition there will be required

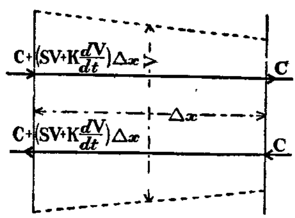


FIG. 4.

a quantity of electricity to satisfy the electrostatic capacity of the length Δx, and this quantity, expressed as a current strength, is equal to $K\,\Delta\,x\,\frac{dV}{dt}$. If C be the current at the right end of the short length of loop, then that at the left end will be $C + \left(S\,V + K\,\frac{dV}{dt}\right)\Delta\,x$. The decrease of current strength from A towards B (see fig. 2), or the increase from B to A due to the combined effects of leakage and capacity, will therefore be $\left(S\,V + K\,\frac{dV}{dt}\right)\Delta\,x$. Again, as $\Delta\,x$ is extremely small, no grave error will arise if it be assumed that the instantaneous current strength in the wires of the short length of loop is C, and that Δ C represents the small change in current strength between the ends of Δx. In which case and considering B to A—

$$\Delta \text{C} = \left(\text{S V} + \text{K} \frac{d\text{V}}{dt}\right) \Delta x,$$

$$\frac{\Delta \text{C}}{\Delta x} = \text{S V} + \text{K} \frac{d\text{V}}{dt} \text{ nearly,}$$

and if, as before, Δx be made less than any assignable magnitude, then, absolutely—

$$\frac{d\text{C}}{dx} = \text{S V} + \text{K} \frac{d\text{V}}{dt}. \quad . \quad . \quad . \quad . \quad . \quad (2)$$

It is from the equations (1) and (2) that the whole theory of telephonic transmission, so far as lines, at least, are concerned, is built up.

It is well known that in any circuit where the impressed voltage is periodic, and a simple harmonic, if V represent the instantaneous value of the voltage after time t, and if V_m represent the maximum value to which the voltage rises, then—

$$\text{V} = \text{V}_m \sin pt,$$

and

$$\frac{d\text{V}}{dt} = p\,\text{V}_m \cos pt,$$

but

$$p\,\text{V}_m \cos pt = p\,\text{V}_m \sin (90 + pt),$$
$$= ip\,\text{V}_m \sin pt,$$
$$= ip\,\text{V},$$

where i represents $\sqrt{-1}$ and indicates the rotation of the quantity to which it is attached through 90°. So that—

$$\frac{d\text{V}}{dt} = ip\text{V}.$$

Similarly it may be shown that—

$$\frac{d\text{C}}{dt} = i\,p\,\text{C}.$$

Inserting these values in the equations (1) and (2) for $\frac{d\text{V}}{dx}$ and $\frac{d\text{C}}{dx}$ respectively—

$$\frac{d\,\mathrm{V}}{dx} = \mathrm{C\,R} + i\,p\,\mathrm{L\,C},$$

$$\frac{d\,\mathrm{C}}{dx} = \mathrm{V\,S} + i\,p\,\mathrm{K\,V},$$

or,

$$\frac{d\,\mathrm{V}}{dx} = (\mathrm{R} + i\,p\,\mathrm{L})\,\mathrm{C}, \quad . \quad . \quad . \quad . \quad (3)$$

$$\frac{d\,\mathrm{C}}{dx} = (\mathrm{S} + i\,p\,\mathrm{K})\,\mathrm{V}. \quad . \quad . \quad . \quad (4)$$

The quantity $\mathrm{R} + i\,p\,\mathrm{L}$ is an *impedance*, and if, as stated above, i indicates the rotation of magnitude through 90°, then the expression may be represented by a right-angled triangle as shown in fig. 5, where I denotes impedance; so that—

$$\mathrm{I} = \mathrm{R} + i\,p\,\mathrm{L},$$

or

$$\mathrm{I} = \sqrt{\mathrm{R}^2 + p^2\,\mathrm{L}^2}.$$

The quantity $\mathrm{S} + i\,p\,\mathrm{K}$ is the reciprocal of an impedance, and is called the *admittance*. It also may be represented by

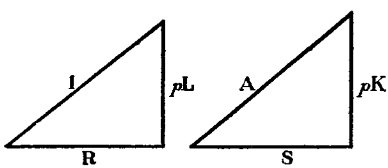


Fig 5.

a right-angled triangle, as shown to the right in fig. 5, where it is indicated by A. In this case—

$$\mathrm{A} = \mathrm{S} + i\,p\,\mathrm{K},$$

or

$$\mathrm{A} = \sqrt{\mathrm{S}^2 + p^2\,\mathrm{K}^2}.$$

Putting

$$\mathrm{I} = \mathrm{R} + i\,p\,\mathrm{L},$$

and

$$\mathrm{A} = \mathrm{S} + i\,p\,\mathrm{K}$$

in the equations (3) and (4)—

$$\frac{d\,\mathrm{V}}{dx} = \mathrm{I\,C}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

and

$$\frac{d\,\mathrm{C}}{dx} = \mathrm{A\,V}. \quad . \quad . \quad . \quad . \quad (6)$$

These equations, (5) and (6), show, for any element of the loop, how the difference of potential between the wires varies, and how the current strength changes. To determine how the voltage and current strength vary with the distance from one end of the circuit, it is necessary to differentiate again with respect to distance. It will be convenient to take any distance, x, from the receiving end of the circuit—say, that shown in fig. 2—because, as this distance increases, both voltage and current strength increase, and the use of negative signs is avoided. Thus—

$$\frac{d^2 \mathrm{V}}{dx^2} = \mathrm{I} \frac{d \mathrm{C}}{dx},$$

$$\text{and } \frac{d^2 \mathrm{C}}{dx^2} = \mathrm{A} \frac{d \mathrm{V}}{d x}.$$

Inserting the values for $\frac{d \mathrm{V}}{dx}$ and $\frac{d \mathrm{C}}{dx}$ shown in equations (5) and (6)—

$$\frac{d^2 \mathrm{V}}{dx^2} = \mathrm{I\,A\,V},$$

$$\frac{d^2 \mathrm{C}}{dx^2} = \mathrm{I\,A\,C}.$$

Let $\mathrm{I\,A} = a^2$,

$$\text{then } \frac{d^2 \mathrm{V}}{dx^2} = a^2 \mathrm{V},$$

$$\text{and } \frac{d^2 \mathrm{C}}{dx^2} = a^2 \mathrm{C}.$$

Both of these equations are of the form—

$$\frac{d^2 y}{dx^2} = a^2 y,$$

and from earlier considerations it is concluded that the solutions are—

$$\mathrm{V} = \mathrm{M}\, \epsilon^{ax} + \mathrm{N}\, e^{-ax},$$

$$\mathrm{C} = \mathrm{M}_1\, e^{+ax} + \mathrm{N}_1 e^{-ax}.$$

As both V and C vary in the same way, it is only necessary to consider one of them. Let it be V.

Taking a particular point in the circuit, it will be seen that the nearer it is to the *sending* end, or the greater x becomes, the greater will be the term $\mathrm{M}\, \epsilon^{ax}$, and the less will be $N e^{-ax}$. The latter term, in fact, represents the reflected wave or impulse from the receiving end. It will be assumed that x is so great that $N e^{-ax}$ becomes negligible or, in the alternative, that there is no reflection; in which case—

$$V = M \epsilon^{ax}.$$

All the following results will, therefore, be understood to apply only to the case where the whole energy received at the end of the circuit is absorbed. In Part III will be found the description of a means of practically determining what is lost by terminal reflections, &c.

The distance x has been taken as that between the element under consideration and the receiving end of the circuit. To express the results, already obtained, in terms of the distance from the sending end, it is only necessary to put $x = m - d$ where—

m = total length of circuit in miles,

d = distance of the point from sending end, in miles.

Taking $$V = M \epsilon^{ax},$$

then $$V = M \epsilon^{a(m-d)},$$

$$= M \epsilon^{am} . \epsilon^{-ad}.$$

Now, at the sending end itself, $d = 0$, so that V then equals $M \epsilon^{am}$. In other words, $M \epsilon^{am}$ is the voltage at the sending end. Let this be represented by V_0; then—

$$V = V_0 \epsilon^{-ad} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

It has been stated that $a^2 = I A$, so, therefore, $a = \sqrt{I A}$. But any quantity such as a^2 may be written equal to the sum of two other squares; thus—

$$a^2 = \beta^2 + \alpha^2 = I A,$$

and since α is at right angles to β,

$$a = \beta + i \alpha.$$

Inserting this value for a in equation (7)—

$$V = V_0 \epsilon^{-(\beta + i \alpha) d},$$

$$= V_0 \epsilon^{-\beta d} . \epsilon^{-i \alpha d} \quad . \quad . \quad . \quad (8)$$

The factor $\epsilon^{-\beta d}$ represents a decaying quantity, and indicates the way in which the difference of potential between the wires of the loop decreases with the distance d from the sending end; but $e^{-i\alpha d}$ is of a different nature. By De Moivre's theorem—

$$e^{-i\alpha d} = \cos\alpha d - i\sin\alpha d,$$

$$\text{so that } V = V_0 e^{-\beta d}(\cos\alpha d - i\sin\alpha d).$$

It is known that any quantity multiplied by $\cos\alpha d + i\sin\alpha d$ is advanced by an angle αd in a positive direction, and that one multiplied by $\cos\alpha d - i\sin\alpha d$ is retarded or turned through an angle αd in a negative direction. The meaning of $V = V_0 e^{-\beta d}(\cos\alpha d - i\sin\alpha d)$ is, therefore, that the difference of potential, V, between the wires of the loop at distance d from the sending end is $V_0 e^{-\beta d}$ where V_0 is the difference of potential at the sending end; and V lags behind V_0 by an angle αd.

It is scarcely necessary to emphasise the fact that the greater the distance d, the greater will be the lag. It will also be clear that, as the distance from the sending end is increased, a point will be reached at which the voltage between the wires is one complete cycle or period behind that impressed at the sending end. Now, from the fact that the phase changes uniformly with the distance, it follows that the waves are travelling along the circuit with a constant velocity, and if at distance d the phase has been delayed by one period, then—

$$\alpha d = 2\pi,$$

and

$$d = \frac{2\pi}{\alpha}.$$

Suppose that f complete periods have been passed through by the E.M.F. at the sending end, then $f - 1$ periods will have been passed through by the difference of potential at the distance d where the phase is one cycle in arrear. Obviously, the time taken for a wave to reach the point at distance d is the same as that occupied by one period. The velocity, v, of the waves is, therefore, shown by—

$$v = \frac{d}{t} = \frac{\frac{2\pi}{\alpha}}{t},$$

where t is the time of one period. If there be f periods per second, then $t = \frac{1}{f}$.

So that $$v = \frac{\frac{2\pi}{a}}{\frac{1}{f}} = \frac{2\pi f}{a},$$

also wave length $$= \lambda = \frac{\text{velocity}}{\text{periodicity}},$$

$$\lambda = \frac{\frac{2\pi f}{a}}{f} = \frac{2\pi}{a}.$$

From the immediately preceding consideration, it will be gathered that a is a constant which determines the velocity with which the waves travel along the circuit, and from equation (8) it will be seen that β is a constant determining the attenuation of the waves at any distance d from the sending end.

a is therefore called the velocity constant.
β „ „ attenuation „

It has been stated that—

$$a^2 = \beta^2 + a^2 = \text{I A},$$

where $$\text{I} = \sqrt{\text{R}^2 + p^2 \text{L}^2},$$

and $$\text{A} = \sqrt{\text{S}^2 + p^2 \text{K}^2},$$

therefore $$\beta^2 + a^2 = \sqrt{(\text{R}^2 + p^2\text{L}^2)(\text{S}^2 + p^2\text{K}^2)}. \quad . \quad . \quad . \quad (9)$$

Also $$a = \beta + i\,a = \sqrt{\text{I A}},$$
$$a^2 = \beta^2 - a^2 + 2\,i\,a\,\beta = \text{I A},$$

where $$\text{I} = \text{R} + i\,p\,\text{L},$$

and $$\text{A} = \text{S} + i\,p\,\text{K},$$

therefore, $$\beta^2 - a^2 + 2\,i\,a\,\beta = (\text{R} + i\,p\,\text{L})(\text{S} + i\,p\,\text{K}),$$
$$= \text{R S} - p^2\,\text{L K} + i\,p\,(\text{K R} + \text{S L}).$$

In an equation like this, the real and unreal quantities on the one side are equal to the real and unreal on the other, respectively; so that—

$$\beta^2 - a^2 = \text{R S} - p^2\,\text{L K}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

By subtracting equation (10) from (9)—

$$2\, a^2 = \sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)} - (R S - p^2 L K),$$

$$a = \sqrt{\tfrac{1}{2}\left[\sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)} - (R S - p^2 L K)\right]}, \quad (11)$$

which is the expression for the velocity constant.

By adding together equations (9) and (10)—

$$2\, \beta^2 = \sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)} + (R S - p^2 L K),$$

$$\beta = \sqrt{\tfrac{1}{2}\left[\sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)} + (R S - p^2 L K)\right]}, \quad (12)$$

which is the expression for the attenuation constant.

These expressions for the velocity and attenuation constants were given by Mr. Oliver Heaviside; see page 400, Vol. II, "Electrical Papers," Macmillan, 1892.

PART II.

In this section of the paper, it is proposed to deal with the application of the formulæ for a and β to practical cases, and the simplification of the expressions where possible. It will be convenient to divide telephone circuits, so far as the lines themselves are concerned, into two classes: first, those in which the inductance is merely that between the ordinary wires forming the loop, which circuits are said to be *unloaded*, and, second, those in which the inductance has been increased by distributing iron along the whole length of the conductors, or in which inductance coils have been inserted at regular intervals. Circuits of this second class are said to be *loaded*; and coils used to increase the inductance are known as *loading coils.*

Before proceeding further, it is necessary to emphasise that any sound made in normal speech is complex; inasmuch as it may be composed of a number of superposed vibrations or waves, some having periodicities as low, or lower, than 100 per second, and others as high, or higher, than 2,000 per second. These vibrations or waves should be reproduced in their proper order or proportion in a telephone circuit if good speech transmission is to be obtained. Supposing all the sound waves at the sending end to have been properly converted into periodic currents of electricity, then, if all the variations of current strength reached the receiving end of the circuit in their proper order and proportion, the reproduction of the quality of the original speech—or articulation, as it is called—would be perfect. An inspection of the formulæ for both α and β will show that they each become greater with the frequency, which means that waves of different frequencies are not transmitted with the same velocity, nor are they subject to the same attenuation. If, however, K R be made equal to S L, then the expression for α reduces to—

$$\alpha = p \sqrt{\text{K L}} = 2 \pi f \sqrt{\text{K L}}.$$

Now the velocity, V, $= \dfrac{2 \pi f}{\alpha}$

$$= \frac{2 \pi f}{2 \pi f \sqrt{\text{K L}}} = \frac{1}{\sqrt{\text{K L}}},$$

so that, if K and L were independent of frequency, all waves would travel with the same velocity. Again, on the same assumption, namely, that K R = S L, the equation for β reduces to—

$$\beta = \sqrt{\text{R S}},$$

which is apparently independent of the frequency; and, if this be the case, all waves are proportionately attenuated. When these conditions obtain, the circuit is said to be *distortionless*. In the first place, no ordinary substance used as a dielectric, other perhaps than air, is known to remain fairly constant in specific inductive capacity and leakance for all telephonic frequencies; while, if there be any iron in the neighbourhood of the conductor, it is improbable—if not impossible—that R and L will remain constant for all frequencies; so that the distortionless circuit, so far as the telephone engineer is concerned, is a thing at present impossible of attainment. Even supposing it were easy to attain, say, by increasing L or reducing S and taking into consideration the absolutely lowest present known values for K and R, in any case of practical telephony, it would be found that the resulting attenuation would be so great, although the articulation might be perfect, that the volume or loudness of speech obtained would be too small on a lengthy line to be of any commercial use whatever.

As the attenuation is different for different frequencies, it is necessary to state in every case what periodicity has been assumed. It is usual to take an average frequency for telephonic currents, and to regard all the elementary waves making up a complex wave as being attenuated in thesame way as the average. What this mean value is, is a matter of some debate. Generally, in this country, it is taken as 750 periods per second; but some place it as high as 800. For attenuation calculations the former will be assumed unless otherwise stated. Later on, it is explained why, when velocity is being considered, 2,000 is taken as the periodicity.

Unloaded Lines.—Although underground circuits, and the like, are primarily being considered, it may be mentioned in passing that in calculating the attenuation of an ordinary overhead line the full formula for β should be used, as none of the factors are negligible. In underground or submarine cable circuits, however, the insulation resistance between wire and wire is of a high order; rarely being as low as 5 megohms per mile, and often as high as 2,000. Generally, then, S, the leakance, is not greater than 0·0000002 mhos; so that, without producing any appreciable error in the results, S may be neglected. If this be done—

$$\alpha = \sqrt{\tfrac{1}{2} p \mathrm{K} (\sqrt{\mathrm{R}^2 + p^2 \mathrm{L}^2} + p \mathrm{L})},$$

$$\beta = \sqrt{\tfrac{1}{2} p \mathrm{K} (\sqrt{\mathrm{R}^2 + p^2 \mathrm{L}^2} - p \mathrm{L})}.$$

These formulæ in cases where S is negligible are applicable to all unloaded subterranean and submarine telephone circuits possessing normal resistance, inductance and electrostatic capacity. A further simplification is possible in special cases, such as when the resistance of the circuit is high compared with the product p L. The inductance of any loop in an underground paper cable, unloaded, may be taken as 0·001 henry per mile; consequently, if $p = 2\pi \times 750$, $p \mathrm{L} = 4·7$ approximately. In the case of 10 or 20 lb. per mile copper conductor loops, where the resistance per mile of loop is 176 and 88 ohms respectively, it may be taken that $\sqrt{\mathrm{R}^2 + p^2 \mathrm{L}^2} = \mathrm{R}$ nearly, and that $\sqrt{\mathrm{R} - p \mathrm{L}} = \sqrt{\mathrm{R}}$ nearly, with the result that—

$$\beta = \sqrt{\tfrac{1}{2} p \mathrm{K} \mathrm{R}}.$$

It should, however, be understood that the result is only approximately correct, say, to within 5 per cent.

Loaded Lines.—In the case of circuits in which the inductance per mile of loop has been greatly increased *by the insertion of loading coils*, a considerable simplification of the formulæ for α and β is possible. It should be noted that the following consideration does not apply, at present, to cases where the inductance is increased by winding iron wire over the conductors. The reasons for making this reservation are given in Part III.

In a properly loaded circuit the product p L is much greater than R; and in any normal underground or submarine circuit the product p K is much greater than S.

Take, first, the expression—

$$\alpha = \sqrt{\tfrac{1}{2}\left[\sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)} - (R S - p^2 L K)\right]}$$

and extract from it $\sqrt{R^2 + p^2 L^2}$.

$$\sqrt{R^2 + p^2 L^2} = p L \left(1 + \frac{R^2}{p^2 L^2}\right)^{\frac{1}{2}},$$

and expanding this by the binomial theorem—

$$\sqrt{R^2 + p^2 L^2} = p L \left(1 + \frac{R^2}{2 p^2 L^2} + \frac{R^4}{8 p^4 L^4} \ldots \&c.\right);$$

all terms involving higher powers than the square may be ignored as being too small to make any practical difference in the result, so that—

$$\sqrt{R^2 + p^2 L^2} = p L \left(1 + \frac{R^2}{2 p^2 L^2}\right) \text{ nearly.}$$

Similarly, it may be shown that—

$$\sqrt{S^2 + p^2 K^2} = p K \left(1 + \frac{S^2}{2 p^2 K^2}\right) \text{ nearly.}$$

The term $\sqrt{(R^2 + p^2 L^2)(S^2 + p^2 K^2)}$

then becomes $\left[p L \left(1 + \frac{R^2}{2 p^2 L^2}\right) . \ p K \left(1 + \frac{S^2}{2 p^2 K^2}\right)\right]$

or $$p^2 K L \left(1 + \frac{R^2}{2 p^2 L^2} + \frac{S^2}{2 p^2 K^2} + \frac{S^2 R^2}{4 p^4 L^2 K^2}\right).$$

The last term within the bracket is so small a quantity as to be negligible, and by multiplying out the expression becomes—

$$p^2 K L + \frac{p^2 R^2 K L}{2 p^2 L^2} + \frac{p^2 S^2 K L}{2 p^2 K^2}$$

$$= p^2 K L + \frac{R^2 K}{2 L} + \frac{S^2 L}{2 K}.$$

The complete expression for α then becomes—

$$\alpha = \sqrt{\tfrac{1}{2}\left[p^2 \text{K L} + \frac{\text{R}^2 \text{K}}{2 \text{L}} + \frac{\text{S}^2 \text{L}}{2 \text{K}} - \text{R S} + p^2 \text{L K}\right]}, \quad (13)$$

$$= \sqrt{p^2 \text{K L} + \frac{\text{R}^2 \text{K}}{4 \text{L}} + \frac{\text{S}^2 \text{L}}{4 \text{K}} - \frac{\text{R S}}{2}}$$

$$= p \sqrt{\text{K L} \left(1 + \frac{\text{R}^2}{4 p^2 \text{L}^2} + \frac{\text{S}^2}{4 p^2 \text{K}^2} - \frac{\text{R S}}{2 p^2 \text{K L}}\right)}$$

$$= p \sqrt{\text{K L} \left[1 + \left(\frac{\text{R}}{2 p \text{L}} - \frac{\text{S}}{2 p \text{K}}\right)^2\right]}.$$

If, as stated, p L be great compared with R, and p K be great compared with S, then—

$$\frac{\text{R}}{2 p \text{L}} - \frac{\text{S}}{2 p \text{K}} \text{ is negligible,}$$

and $$\alpha = p \sqrt{\text{K L}},$$

which result is *not* to be compared with the one of similar form given previously, because in this case it is *not* assumed that K R = S L.

Take next the expression for β, and notice that it only differs from that for α in that the last term under the principal root sign is positive. Making the same assumption as before in connection with the magnitudes of p L and p K, the complete expression for β is (from equation 13)—

$$\beta = \sqrt{\tfrac{1}{2}\left[p^2 \text{K L} + \frac{\text{R}^2 \text{K}}{2 \text{L}} + \frac{\text{S}^2 \text{L}}{2 \text{K}} + \text{RS} - p^2 \text{L K}\right]}, \quad (14)$$

$$= \sqrt{\frac{\text{R}^2 \text{K}}{4 \text{L}} + \frac{\text{S}^2 \text{L}}{4 \text{K}} + \frac{\text{R S}}{2}}$$

$$= \sqrt{\frac{\text{R}^2 \text{K}}{4 \text{L}} \left(1 + \frac{\text{S}^2 \text{L}^2}{\text{K}^2 \text{R}^2} + \frac{2 \text{S L}}{\text{K R}}\right)}$$

$$= \sqrt{\frac{\text{R}^2 \text{K}}{4 \text{L}} \left(1 + \frac{\text{S L}}{\text{K R}}\right)^2}$$

$$= \frac{\text{R}}{2} \sqrt{\frac{\text{K}}{\text{L}}} \left(1 + \frac{\text{S L}}{\text{K R}}\right)$$

$$= \left(\frac{R}{2} + \frac{S\,L}{2\,K}\right)\sqrt{\frac{K}{L}}$$

$$= \frac{R + \frac{S}{K}\,L}{2}\sqrt{\frac{K}{L}}.$$

This formula gives the attenuation constant for any loaded circuit provided p L be great compared with R, and p K oe great with respect to S, as is usually the case.

Methods of determining the values of R and L at different frequencies are fairly well known, and for the particular periodicity considered, we may, therefore, insert the measured values in the above formula. This, however, is not the case with S and K. Very little information is available respecting the variation of electrostatic capacity and insulation resistance with frequency. Further experimental investigation appears to be necessary on the following points :—

(*a*) What is the periodicity which may be taken as the mean of a speech wave ?

(*b*) Is this mean frequency the same for all types of circuit, and if not, how does it vary ?

(*c*) How do electrostatic capacity and leakance vary with frequency in paper and gutta-percha-covered conductors such as are commonly used in practice?

In this paper it is not intended to discuss these matters, owing to lack of information of a definite character. There is, however, one assumption which may be made with some degree of safety : namely, that at any particular frequency the ratio of S to K will be constant.

Let $$\frac{S}{K} = n,$$

then $$\beta = \frac{R + n\,L}{2}\sqrt{\frac{K}{L}}.$$

In the present incomplete state of knowledge as to the variation of K with frequency and of what the mean frequency in any particular circuit is, the only thing to be done is to take K as being the same at all periodicities, and by associating a large number of practical results to fix a value to the constant n, which most nearly fits the observed results. This, as will be seen later on, has been done, with the results that when the loading coils used have non-magnetic cores, and have no magnetic material near them,

$n = 50$; and, when the loading coils have cores of magnetic material in the form of a closed magnetic circuit, then, $n = 80$, give results sufficiently near the truth for present practice in the matter of estimating what improvement will be effected by loading a particular circuit. It must, however, be noted that these figures are based upon the observations made with the coils described in Part III ; where it is mentioned that the effective resistance of so-called "iron-core coils" is much less than that of the "air-core coils" for a given inductance. The difference in effective resistance of the two types of coil for the same inductance may explain the reason for the different value of the constant n required to make the observed and calculated results agree. These matters require further thought and investigation, but until such investigation has been made, it may be accepted with confidence that—

(*a*) Where loading coils *without* magnetic ~~coils~~ cores are used,

$$\beta = \frac{R + 50\,L}{2} \sqrt{\frac{K}{L}}.$$

(*b*) Where loading coils with magnetic cores (such as are mentioned hereafter) are employed,

$$\beta = \frac{R + 80\,L}{2} \sqrt{\frac{K}{L}}.$$

Anyone who studies these formulæ, or, indeed, who scans the practical results given in Part III, will at once appreciate the necessity for keeping the resistance R as low as possible for any given increase in the value of the inductance L.

It is very lamentable that little attention appears to have been paid to Mr. Oliver Heaviside's repeated indications that inductance would be beneficial in a telephone loop. Some eminent scientists appear to have recognised the importance of the matter, amongst them being Prof. S. P. Thompson, but no definite information as to the manner in which the inductance might be efficiently inserted appears to have been given until the subject was investigated mathematically and experimentally in America by Prof. M. I. Pupin, whose papers are worthy of the most serious consideration ; they are "Transactions of the American Institute of Electrical Engineers," (*a*) March 22nd, 1899 ; "Propagation of Long Electrical Waves," (*b*) May 19th, 1900 ; "Wave Transmission over Non-Uniform Cables and Long-Distance Air Lines."

Other valuable work has also been done by Dr. G. A. Campbell, see *Phil. Mag.*, 1903, Vol. 5, page 313, "On Loaded Lines in Telephonic Transmission."

Lagrange in his "Mecanique Analytique," Sec. partie, Sec. VI, Lord Rayleigh in his "Theory of Sound," Vol. I, pages 233 and 234, Prof. Routh in his "Advanced Rigid Dynamics," page 260, and many others have investigated the propagation of waves along uniformly loaded strings. They show, in effect, that if a long, thin stretched string be taken and loaded by small weights at equal distances along

Loaded string

FIG. 6.

its length, it will vibrate practically as though uniformly loaded if there be not less than π loads per wave length ; or, that the string will transmit waves when the length of each wave is such as to include π loads. Of course, the closer the loads, the shorter will be the wave length which the string will be able to transmit.

Prof. Pupin and Dr. Campbell each seem to have recognised an analogy between the behaviour of a loaded string under tension, fig. 6, and that of an electric circuit, fig. 7, where the inductance coils inserted at regular intervals take the place of the loads on the string. The papers of the able investigators referred to will throw all necessary light upon this matter. Because the inductances inserted in an electric

Loaded single wire

Loaded loop

FIG. 7.

circuit limit the wave length which can be transmitted, in the same way as the loads upon a string limit the wave lengths which it will transmit, they are regarded as loads and the coils are called *loading coils.*

Working upon this analogy between the transmission of waves along a loaded string under tension, and a loaded electric circuit respectively, it will be accepted that there must be at least π coils per wave length in the circuit. To obtain sufficiently clear speech for practical purposes, it is essential that waves having a periodicity of 2,000 per second should be transmitted; there must, therefore, be π coils per wave length at this high frequency. This fact may be made use of in establishing a simple rule for the minimum distance apart for the loading coils selected to serve any circuit. Thus—

$$\text{Wave length} = \lambda = \frac{2\pi}{a}.$$

If D be the distance in miles between the coils, then—

$$D = \frac{\lambda}{\pi} = \frac{2}{a}.$$

But in a loaded circuit it has been shown that—

$$a = p\sqrt{KL} = 2\pi f\sqrt{KL}$$

where f is the periodicity per second.

$$\therefore\ D = \frac{2}{2\pi f\sqrt{KL}}$$

$$= \frac{1}{\pi f\sqrt{KL}}.$$

and

$$D^2 = \frac{1}{\pi^2 f^2 KL}$$

Now let K_1 and L_1 be the electrostatic capacity in *micro-farads* and the inductance in *millihenrys* respectively per mile of loop. Then—

$$D^2 = \frac{10^9}{\pi^2 f^2 K_1 L_1}.$$

But the inductance between the actual wires of any telephone loop in practice is so small as to be negligible compared with that added in loading the circuit; so that L_1 may be taken as the inductance per mile due to the coils. If, therefore, L_2 be the inductance in *millihenrys* of each coil used—

$$L_1 = \frac{L_2}{D},$$

and by substituting this in the above equation for D^2—

$$D = \frac{10^9}{\pi^2 f^2 K_1 L_2},$$

or

$$D K_1 L_2 = \frac{10^9}{\pi^2 f^2}.$$

Taking $f = 2{,}000$ periods per second—

$$D K_1 L_2 = 25\cdot3.$$

This result represents the worst permissible condition—namely, that in which there are just π coils per wave at 2,000 periodicity. It is better in practice to allow some margin, so that commonly πf is taken as equalling 7,000, which, according to the view taken, allows more than π coils per wave at 2,000 periods, or allows shorter waves to be transmitted at π coils per wave-length. If $\pi f = 7{,}000$, then—

$$D K_1 L_2 = 20\cdot4.$$

The general rule is, therefore, that the product of the distance apart of the coils in miles, the electrostatic capacity per mile in microfarads, and the inductance per coil in millihenrys should not be greater than 25·3, and need not be less than 20·4.

PART III.

THE following known methods of loading a telephone loop have been mentioned :—

(*a*) *Coil Loading*, consisting of the insertion of inductance coils at regular intervals in the circuit ;

(*b*) *Continuous Loading*, consisting of the provision of one or more coatings of iron wire, or other magnetic material, upon each wire of the loop.

Coil loading will be first considered, but, before proceeding to state the practical results obtained and the method of determining the improvement due to loading, it may be desirable to describe briefly the coils used and the method of including them in cable circuits.

In the first instance, before the development of the present-day magnetic-core loading coil, experiments were made with inductances containing no iron or any other magnetic material. The most recent form of such a loading coil is illustrated by fig. 8. It consists of a bobbin of insulating material, such as ambroin, divided into two equal parts by a diaphragm. Two similar coils of silk-covered

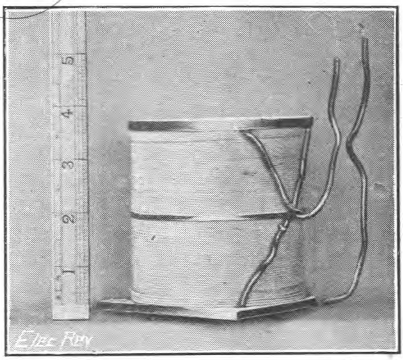


FIG. 8

copper wire are wound, one upon each side of the diaphragm. There will, of course, be four ends to each loading coil, but only two are clearly visible in the illustration. The general dimensions may be taken by reference to the inch scale to

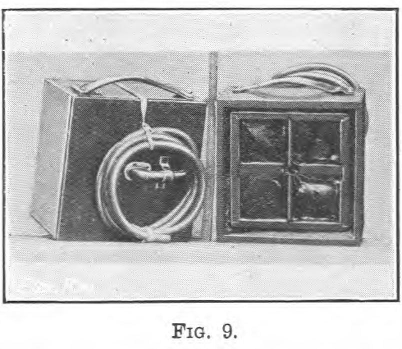


FIG. 9.

the left of the figure. Four of these bobbins are then placed in a box of teak or other hard wood, so that the axes of three of the coils are at right-angles to each other; the fourth coil being placed with its axis nearly parallel with that of the one diagonally situated with respect to it in the containing box, and the whole are adjusted until no inductive effect takes place between any one and the others. The box is then filled with compound and placed in a second hard-wood case, the space between the boxes being filled carefully with petroleum jelly. The coil ends are jointed to, and led out of the case by means of, a silk and cotton-insulated copper wire, lead-covered cable; the whole presenting the appearance shown by fig. 9.

By the courtesy of the Western Electric Co., Ltd., an illustration of a modern magnetic-core loading coil is given in fig. 10, from which the general dimensions may be estimated. The core is a ring, approximately circular in

FIG. 10.

cross-section, made up of finely drawn wire of some magnetic material, the exact description of which cannot be given. Upon this ring two similar coils of insulated copper wire are wound, and the whole is steeped in highly insulating wax. Any desired number of such coils are then placed in a strong, cast-iron, air-tight case similar to those shown in fig. 11; the coil ends being connected to, and brought out of the case by means of, india-rubber and cotton-covered lead-sheathed cable.

The above-mentioned cases of coils are only used on underground cables. At the points along the route of the cable at which it has been decided to insert coils in the circuits, a brick-built pit such as a joint box, chamber, or manhole, is provided and the cases of coils are stored therein. The method of connecting the coils to the main cable is the same for both air-core coils in wooden cases and iron core coils in iron cases. Within the pit or manhole is fixed a cable distribution head (fig. 12), which consists of a cast-iron box and ribbed lid capable of being made air-tight when combined. The main cable from one direction is led

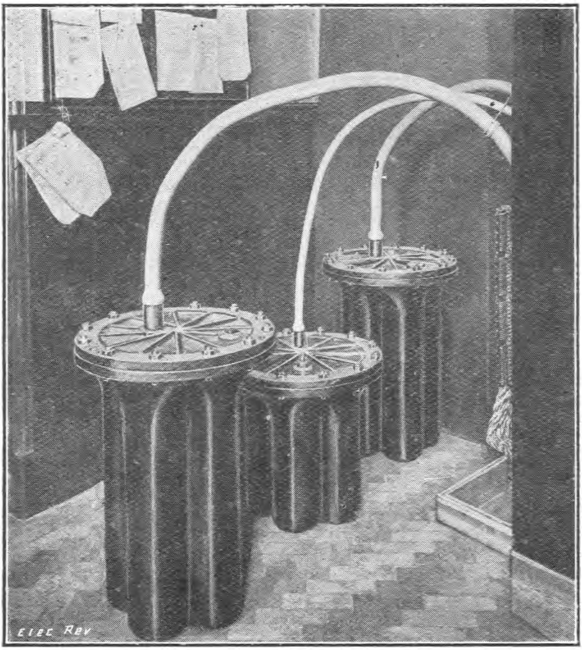


FIG. 11.

into the box or head at A and the cable from the other direction at B; while the lead-covered cable from the coil case enters the head at any point in either side—say C, fig. 12. The cables at A and B are connected together through the coils, the ends of which are contained in cable C. If more than one case of coils be necessary for the loading of the cable, additional coil-ends may be led into the box at such positions as D, E and F.

In describing the method of arranging air-core coils in wooden cases, it was mentioned that they were specially arranged so as to eliminate inductance among themselves. Every air-core coil has an external electromagnetic field; consequently there are the following disadvantages :—(*a*) If the coils be placed near any mass of metal, eddy currents are set up in the latter, which, by reaction on the coil, increase its *effective* resistance and lower its efficiency; (*b*) If a case of air-core coils be placed near a mass of iron, the electromagnetic fields of the coils are distorted; the conditions under which they were arranged in the case so as to be free from inductive disturbance, one upon another, do not hold, and overhearing occurs between all of them. (*c*)

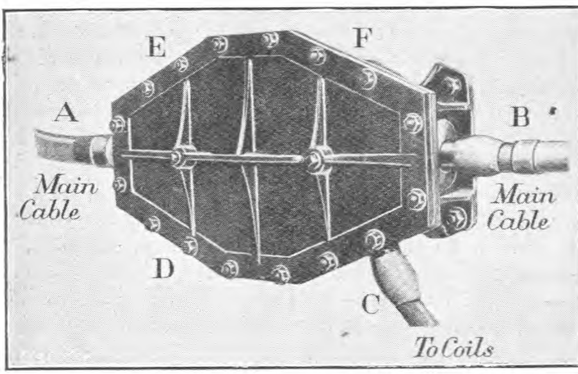


FIG. 12.

Two similar boxes of coils cannot be placed near to and parallel with each other without the coils of one being parallel with those of the other; in these circumstances, overhearing must ensue between the circuits passing through the coils in the two boxes.

No means are at present known of overcoming the disadvantages (*a*) and (*b*), but that mentioned in (*c*) is overcome by constructing two types of case. In one the coils are arranged at right-angles to those in the other. One case is known as "right," the other as "left," and where the air-space type of coil is used, these cases should be arranged alternately in the pits or manholes, and be placed not less than 4 in. apart. Distinct from the foregoing disadvantages of the air-core coil are the following :—(*d*) The effective resistance for a given inductance is high; (*e*) owing to the impossibility of encasing such coils in metal, it is difficult to make and maintain the cases in which they are placed air and water-tight—conditions necessary in the upkeep of a modern underground cable.

In the use of the so-called iron-core coils there are no such disadvantages as those just mentioned. The coils, being wound upon rings of magnetic material, have no, or negligible, external magnetic fields; consequently they may be encased in metal, and, provided a thin iron shielding plate be placed between them as a precaution against electrostatic or slight electromagnetic effect, may be placed one upon another to any extent without being subject to inductive interference between themselves or from other sources. Moreover, since metal may be employed, the cases may be made air and water tight and durable. One of the most important points, however, is that properly designed so-called iron-core coils possess a much higher inductance than air-core coils for a given effective resistance.

Although the expression "iron-core coil" will be used in future, it will be understood that the core may not consist of *pure* iron. Such coils have one disadvantage—namely, that if a current of abnormal strength from a telephone point of view—say, 0·15 ampere—be passed through the circuit for testing or other purposes, the cores of the coils are liable to become permanently magnetised, in which case the inductance of the coils is lowered, and the effective resistance increased; a result naturally followed by a decrease in the efficiency of the circuit. This, however, is not a serious matter, since any suspected permanent magnetisation of the cores of the loading coils may be readily removed by one or two applications of the following treatment:—Apply a relatively strong alternating voltage to the circuit so as to cause a current of 0·2 virtual ampere to pass through it. Then, by inserting resistance in the loop, gradually reduce the current strength to less than 0·001 ampere.

In view of the foregoing facts, there is little doubt but that the coil with a continuous and closed core of some magnetic material will be the loading coil of the future.

There are several ways in which the transmission efficiency of a telephone circuit may be measured, but the most satisfactory in practice, so far, is that known as the method of *standard cable equivalents*. By this method a length of cable, the electrical and attenuation constants of which are known, is arranged so that equal volumes or loudness of speech are received from the same instruments through it and the circuit under trial. The cable used as a standard of reference is called *standard cable*, and in this country consists of a copper conductor, paper insulated, loop circuit enclosed in a lead sheath and possessing the following electrical constants :—

Conductor resistance	88 ohms per mile of loop.
Electrostatic capacity	0·054 mfd. per mile, wire to wire.
Minimum insulation resistance ...	2)0 megohms per mile, wire to wire.

Such a loop possesses an inductance of 0·001 henry per mile, and if this be taken into account in calculating the attenuation constant by means of the formula given for unloaded circuits, the value 0·103 per mile is obtained. That the attenuation constant for standard cable is 0·103 per mile has been verified by many experiments. Any other normal type of circuit may, of course, be selected as a standard provided its attenuation constant be known. In practice artificial cables adjustable by 1-mile steps, so that any value between 1 and 60 miles of standard cable may be inserted, are used for testing purposes. Each mile section of such cable has the same electrical constants as a mile of actual standard cable, and every artificial cable used is calibrated by comparison with an actual standard cable laid underground ; the latter being adjustable, as regards length, between the limits 1 and 100 miles by 1-mile steps.

The standard cable equivalent of any telephone loop may be defined as the length of standard cable having the same telephonic transmission efficiency as the loop under test. The general method of determining these equivalents is illustrated by fig. 13. T and T_1 are ordinary telephone sets sufficiently removed from each other to make it impossible for one speaker to hear the other except through the telephone circuit. When the lever L of the switch S is in the position shown in the figure, the telephone circuit is completed through the adjustable standard cable ; but if moved in the reverse direction, the longer springs to the right of the switch are forced apart, and the longer springs to the left are released, so as to complete the telephone circuit through the loop under test. By this means an instantaneous change-over from one circuit to the other is possible. Supposing the experimenter at T to have control of the switch and standard cable, the one at T_1 counts or speaks in a normal and level tone until the observer at T has so adjusted the length of standard cable that no difference in the volume or loudness of speech received can be detected whether the standard cable length or the circuit under test be spoken over, *i.e.*, whether the switch be in one position or the other. The result is afterwards checked by speaking

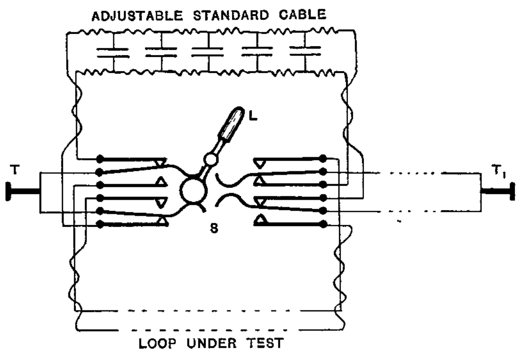


FIG. 13.

or counting similarly from T, while the observer at T_1 listens on each circuit. The length of standard cable through which a volume of speech is received equal to that obtained over the circuit under trial is called the standard cable equivalent of the latter.

It not infrequently happens that the quality of speech—or *articulation*, as it is called—which is received through the loop under test differs slightly from that obtained at the end of the equivalent length of standard cable. Astonishing as it may appear to some, it is nevertheless true that loading produces a much greater effect upon the volume of speech received than it does upon the grade of articulation. Over any economically-loaded loop—namely, one in which there are from π to, say, 5 coils per wave length at 2,000 periods per second—the articulation of the received speech is not so vastly superior to that received over standard cable as to render it difficult, for a telephone engineer possessed of normal hearing, to judge when equality of speech volume is obtained through the loaded circuit and a length of standard cable, when the two loops are compared by the method described in connection with fig. 13.

If the standard cable equivalent of a circuit be known, it is easy to deduce its attenuation constant per mile; thus:—A certain loop 70 miles in length is equivalent to 20 miles of standard cable; what is its attenuation constant per mile? The loop is clearly $\frac{70}{20} = 3{\cdot}5$ times better than standard cable from a telephonic transmission point of view, and its attenuation must therefore be 3·5 less than that of standard cable. If the attenuation constant per mile of standard cable be 0·103, then that of the loop under consideration is $\frac{0{\cdot}103}{3{\cdot}5} = 0{\cdot}03$ nearly. The method of measurement above-mentioned is only applicable in cases where a return metallic loop is available; this is not a disadvantage in practice, since in most telephone cables there are two pairs of wires, and in many there are more than a hundred. This point will be referred to in greater detail later.

Only in exceptional circumstances is the measurement of the standard cable equivalent of *one* particular length of a given type of telephone loop relied upon for the purpose of fixing its transmission efficiency. It is usual to measure as many different lengths of the same type of loop as possible and then plot a curve; the ordinates being actual miles of tested loop and the abscissæ the relative standard cable equivalents. In a cable containing many pairs such different lengths as are required may readily be obtained by joining the loops in series, to and fro, in the cable; *care being taken first to prove that there is no inductive disturbance between the pairs so joined in series.* There are many other points which might be referred to in a general way now; but it will be better to treat them specifically as they arise in the course of describing the practical results obtained. It is proposed to begin with small underground conductors, and afterwards proceed to the heavier ones.

The first case is that of a loop consisting of copper conductors weighing, nominally, 20 lb. per mile. Each conductor has an average diameter of 35·5 mils. The electrical constants of the unloaded loop are as follows:—

Conductor resistance per mile of loop ...	**85 ohms.**
Electrostatic capacity	**0·07 mfd.**
Inductance	**0·001 henry.**
Leakance	**Negligible.**

If the attenuation constant per mile of this loop be calculated from the ordinary formula for unloaded circuits—namely :

$$\beta = \sqrt{\tfrac{1}{2} p \text{K} (\sqrt{\text{R}^2 + p^2 \text{L}^2} - p \text{L})},$$

it will be found to be 0·115 nearly.

The results of actual standard cable measurements on different lengths of the unloaded cable are given in column 2 of Table I, and shown graphically by A in fig. 14.

It will be noticed that the unloaded lengths are only 0·9 times as good as standard cable for telephonic transmission purposes; the attenuation constant is therefore $\frac{0{\cdot}103}{0{\cdot}9} = 0{\cdot}115$ nearly, which agrees with the calculated result given above.

The cable was loaded by inserting iron-core coils at one-mile intervals. Each coil possessed an inductance of 252 millihenries and offered an effective resistance of 10·5 ohms at a frequency of 750 periods per second. The standard cable equivalents of various lengths of the loaded cable loop were then measured; the results being as shown in column 3 of Table I, and as represented by curve B in fig. 14. This

TABLE I.—20-LB. CONDUCTOR LOOP. PAPER CORE CABLE.

Mileage of loop under test	Standard cable equivalent in miles.	
	Circuit unloaded.	Circuit loaded.
10	11·0	6·5
15	—	10·0
20	21·5	11·0
25	—	12·0
30	33·5	14·0
40	44·0	16·5
50	—	19·0
60	—	22·0
70	—	25·5

curve, drawn through the points obtained by experiment, represents the correct relation between the different lengths of loaded loop tested, and their standard cable equivalents. Reading, then, from the curve it is seen that the equivalent of the first 10 miles of loaded loop is 7·5 miles of standard cable, but the addition of any 10-mile length to the loaded loop only increases the standard cable equivalent by 3 miles. The electrical constants of one 10-mile length are identical with those of any other equal length; the true standard cable equivalent of the first 10 miles of loaded circuit is consequently equal to that of any added equal and similar length, namely, 3 miles of standard cable. The value of the first 10-mile length, as shown by the curve, is 4·5 miles of standard cable in excess of the true equivalent of 3 miles.

With the first length of loaded cable loop tested some quantity is measured which is independent of the total length of the loop; such quantity, not being a function of the circuit length, must be dependent upon terminal conditions, *i.e.*, upon the type of apparatus employed. What really happens is

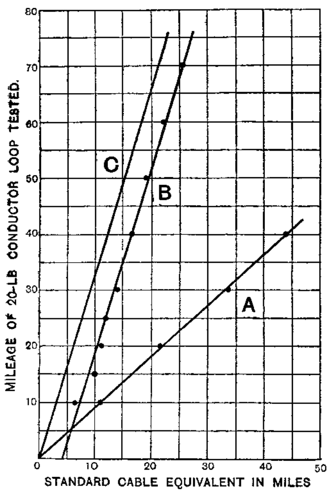


FIG. 14.

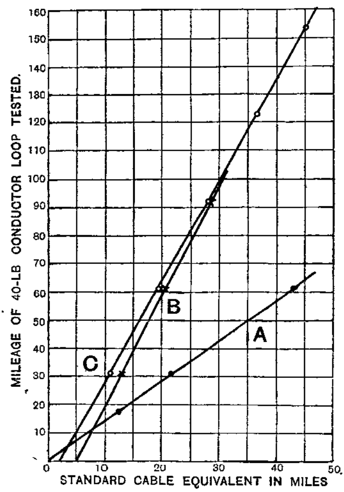


FIG. 15.

that not all the energy received over a loaded circuit is absorbed by telephone apparatus designed for use on unloaded lines; some of the waves are reflected and lost, so far as their effect on the receiving instrument is concerned. In the case of the loaded circuit now under consideration, these reflection losses produce just the same effect as the interposition of 4·5 miles of standard cable in the loop, and, since such loss takes place at the ends of the circuit, it is called *terminal loss*. Later on some particulars of the elimination of terminal loss will be given, but meanwhile it should be noted that the magnitude of such loss is always shown, in terms of standard cable, by the intercept between the origin and the point at which the curve, such as B, fig. 14, strikes the abscissa. Obviously, a curve such as C in the same figure will give the standard cable equivalent of any length of the particular loaded loop exclusive of terminal loss, for C is parallel to B and passes through the origin.

Excluding terminal loss, it will be seen that the loaded circuit, as shown by experiment, is 3·33 times better than standard cable, and its attenuation constant per mile is, therefore, $\frac{0.103}{3.33} = 0.031$ nearly. Now apply the formula—

$$\beta = \frac{R + 80\,L}{2}\sqrt{\frac{K}{L}},$$

which has been given for cases where iron-core coils are used, and bear in mind (1) that the coil resistances at 750 P.P.S. have to be added to the conductor resistance of the loop to obtain the value of R, and (2) that L includes the inductance per mile possessed by the loop—namely, 0·001 henry, as well as that introduced by the coils. Then—

$$\beta = \frac{(85 + 10{\cdot}5) + (80 \times 0{\cdot}253)}{2}\sqrt{\frac{0.07 \times 10^{-6}}{0.253}}$$

$$= 0{\cdot}0304 \text{ per mile.}$$

This value does not differ from that obtained experimentally (0·031) by more than about 2 per cent.

The improvement due to loading is obtained by comparing the attenuation constant of the loaded circuit with that of the same circuit prior to loading. In the present case the ratio is $\frac{0.115}{0.031} = 3{\cdot}7$ nearly. The improvement is 370 per cent.

Wave length (λ) has been shown in Part I to be equal to $\frac{2\,\pi}{a}$; and in Part II, a for loaded circuits was shown to be, as nearly as is necessary for practical calculations, equal to $p\,\sqrt{K\,L}$. For a loaded circuit, therefore—

$$\lambda = \frac{2\,\pi}{p\,\sqrt{K\,L}} = \frac{2\,\pi}{2\,\pi f\,\sqrt{K\,L}} = \frac{1}{f\,\sqrt{K\,L}}.$$

It was also pointed out in Part II that to obtain clear speech there must be at least π coils per wave length at 2,000 periods per second. Applying the wave length formula just given, to the case of the 20-lb. conductor loaded loop under consideration—

$$\lambda = \frac{1}{2{,}000 \sqrt{0{\cdot}07 \times 10^{-6} \times 0{\cdot}253}},$$
$$= 3{\cdot}76 \text{ miles.}$$

As the coils were inserted in the loop at 1-mile intervals, there were, therefore, 3·76 coils per wave as an average. The articulation or quality of speech observed on the circuit was good.

The foregoing examples have been somewhat fully dealt with. It will be understood that the results given hereafter have been arrived at by similar methods.

The next case of interest is that of a loop consisting of copper conductors weighing, nominally, 40 lb. per mile. Each conductor has an average diameter of 50 mils. The electrical constants of this loop, unloaded, were found to be as follows :—

Conductor resistance per mile of loop ...	44 ohms.
Electrostatic capacity	0·055 mfd.
Inductance	0·001 henry.
Leakance	Negligible.

Calculating the attenuation constant per mile of loop from the formula given for unloaded circuits, the value $\beta = 0{\cdot}072$ (nearly) is obtained. Standard cable measurements were taken of different lengths of the loop, the results being those shown in column 2 of Table II, and represented graphically by A in fig. 15.

TABLE II.—40-Lb. Conductor Loop. Paper Core Cable.

Mileage of loop under test.	Standard cable equivalent in miles.		
	Circuit unloaded.	Circuit loaded at 1-mile points.	Circuit loaded at 2-mile points.
17·9	12·6	—	—
30·6	21·6	13·0	11·0
61·2	43·0	20·5	19·5
91·8	—	28·5	28·0
122·4	—	—	36·5
153·0	—	—	45·0

The unloaded loops are therefore 1·42 times better than standard cable ; the attenuation constant per mile is consequently $\frac{0{\cdot}103}{1{\cdot}42} = 0{\cdot}0725$, which value nearly agrees with that calculated.

In the loading of this cable air-core coils were used, each coil having an inductance of 100 millihenries and offering a resistance of 15·4 ohms at 750 P.P.S. These coils were used because they happened to be available, and, with the object of obtaining information, were first tried at 1-mile points in the loops. The results of the tests are given in column 3 of Table II and represented by curve B, fig. 15. The standard cable equivalent of the first length of 30·6 miles of loaded loop is 13 miles, while the mean equivalent of any other equal length is only 7·8 miles of standard cable. The terminal loss is consequently 13 − 7·8 = 5·2 miles of standard cable.

In order to save space let M S.C. represent miles of standard cable, and S.C.E. denote standard cable equivalent.

Since 30·6 miles of the loaded loop are equivalent to 7·8 M.S.C., the former are $\frac{30·6}{7·8}$ = 3·92 times better than standard cable, and the attenuation constant per mile of loaded loop is $\frac{0·103}{3·92}$ = 0·026. Applying the formula for circuits loaded with air-core coils, namely—

$$\beta = \frac{R + 50\,L}{2}\sqrt{\frac{K}{L}},$$

the following result is obtained :—

$$\beta = \frac{(44 + 15·4) + (50 \times 0·101)}{2}\sqrt{\frac{0·055 \times 10^{-6}}{0·101}}$$

$$= 0·024 \text{ nearly, per mile.}$$

The measured attenuation constant of 0·026 is roughly 8 per cent. greater than that given by the formula. This is probably due to the facts (*a*) that the spacing of the coils was irregular to the extent of 10 per cent., although the average distance between coils was 1 mile ; (*b*) that there were about twice as many coils per wave-length as usual.

The wave-length in the loaded circuit will be found to equal 6·7 miles approximately, and with one coil per mile this means 6·7 coils per wave-length, at 2,000 P.P.S. The numerical coefficients of L, namely, 50 for air-core and 80 for iron-core coils, given in the formulæ for the attenuation constant are really meant to apply to cases where the average number of coils per wave does not greatly exceed π. In the above case there are more than 2 π coils per wave-length, yet the discrepancy between calculation and observation is not serious.

The attenuation constant of the loop unloaded being 0·0725, and that of the same loop loaded as described being 0·026 per mile, the improvement due to the loading is $\frac{0.0725}{0.026} = 2.8$ nearly, or almost 280 per cent.

In the next trial the same coils were placed in the loops, but the distance between them was increased to an average of 2 miles. The measured standard cable equivalents of various lengths of loop up to 153 miles are given in column 4 of Table II, and shown by curve C in fig. 15.

The results, briefly, are—

First 30 6 miles of loaded loop		=	11 0 M.S.C.
Each " " "	added	=	8 5 "
	Difference = Terminal loss	=	2·5 M.S.C.

The loaded loops were, therefore, $\frac{30.6}{8.5} = 3.6$ times as good as standard cable, so that the attenuation constant per mile is $\frac{0.103}{3.6} = 0.0286$.

Before attempting to calculate the attenuation, remember that the coils are 2 miles apart, and consequently only half the resistance and inductance respectively of each coil have to be added to the electrical constants per mile of unloaded loop. Performing this calculation by the aid of the formula given for use when air-core coils are employed—

$$\beta = \frac{(44 + 7\,7) + (50 \times 0.051)}{2}\sqrt{\frac{0.055 \times 10^{-6}}{0.051}}$$

$$= 0\,0282 \text{ per mile.}$$

This result only differs by 1·4 per cent. from the value 0·0286 given above and deduced from experiment. As the attenuation constant per mile of unloaded cable was 0·0725, the improvement due to the loading at 2-mile points is 253 per cent.

The wave length in this circuit will be found to be 9·44 miles; so that, with one coil every 2 miles, there will be 4·72 coils per wave length at 2,000 P.P.S. It is noteworthy that the quality of speech received over the loops when loaded at 2-mile points was not judged to be inferior to that received when the same circuits were fitted with coils at 1-mile intervals. Other points worthy of notice are :—(*a*) When the circuits were loaded at 1-mile points, so that the total inductance of the loop was 101 millihenries per mile, the terminal loss was 5·2 M.S.C.; but when loaded at 2-mile points, so as to reduce the inductance to 51 millihenries per mile, the terminal loss was only 2·5 M.S.C.; the lighter loading produces the lesser terminal loss. (*b*) The same coils were used for both 2 and 1 mile spacing. When they were placed 2 miles apart, the measured attenuation constant per mile of circuit was 0·0286 ; when placed 1 mile apart, the attenuation constant per mile was observed to be 0·026. The telephonic transmission efficiencies of the two arrangements are as 1 to 1·1, since they are inversely proportional to the attenuation constants. It is clear, therefore, that *in this particular case* the loops are only increased 10 per cent. in efficiency by doubling the number of coils used, and consequently doubling the expense of loading.

The average inductance per mile of loop in the preceding case was changed by inserting the *same coils*, first at 1 and then at 2-mile intervals in the circuit, but in the trials next described the inductance per mile was varied by using *different coils* and keeping the spacing constant. The experiments were made upon a lead-covered cable 12 miles in length, containing 21 pairs of paper-insulated copper conductors, each wire weighing 100 lb. per mile and having a mean diameter of 79 mils. The electrical constants of the unloaded loop are :—

TABLE III.

Mileage of loop under test.	Standard cable equivalent in miles.		
	Circuit unloaded.	Circuit loaded at 2·44-mile points.	
		Coil D.	Coil E.
24	10	8	8
48	20	12	12
72	30	16	16
96	—	20	20
120	—	—	24
144	—	—	28

Conductor resistance per mile of loop	**18 ohms.**
Electrostatic capacity	**0·055 mfd.**
Inductance	**0·001 henry .**
Leakance	**Negligible.**

The attenuation constant as calculated from the ordinary formula for unloaded underground circuits is 0·042 per mile, which indicates that the unloaded 100-lb. conductor loops should be 0·103/0·042 = 2·45 times better than standard cable from a telephone transmission point of view. By doubling to and fro in the 12-mile length of cable, different lengths of circuit were arranged, and their standard cable equivalents measured; the results are given in column 2 of Table III. It will be seen that any 24-mile length of the unloaded loop is equivalent to 10 miles of standard cable; therefore, the former must be 2·4 times better than the latter. The attenuation constant per mile as deduced from the experimental results is consequently 0·103/2·4 = 0·043; a value nearly in agreement with that of 0·042 obtained by calculation.

Some of the pairs in the cable were next loaded with air-core coils of a type known as D, and others with coils of similar make known as type E. The electrical constants of these coils are:—

TYPE D.

Effective resistance	10·8 ohms, at 750 P.P.S.
Inductance...	0·070 henry " "

TYPE E.

Effective resistance	13·4 ohms, at 750 P.P.S.
Inductance...	0·083 henry " "

These coils were used, not because they were considered to be the best possible, but for the reason that they were in stock. The standard cable equivalents of various lengths of the loaded loops are shown in columns 3 and 4 of Table III. It will be observed that the results are identical. In each case—

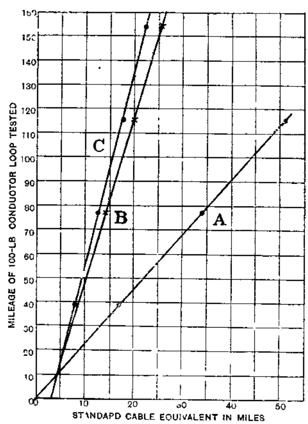
First 24 miles of loaded loop	= 8 M S C.
Each " " " added	= 4 "
Difference = Terminal loss	= 4 M S C.

Either type of loaded circuit was consequently 24/4 = 6 times better than standard cable, and the attenuation constant, therefore, is 0·103/6 = 0·0172 per mile. By calculation from the formula for air-core coils the following values are obtained:—

Coils D. $\beta = 0{\cdot}0162$ per mile.
Coils E. $\beta = 0{\cdot}0158$ "

The circuits loaded with coils D and E, respectively, should be almost equally good, so far as calculation shows, since their attenuation constants only differ by about 2·5 per cent. The beneficial effect produced upon the telephonic transmission efficiency of the loops by the higher inductance of coils E, as compared with those D, is balanced by the adverse effect of the high resistance of the former as compared with that of the latter coils.

The calculated attenuation constant per mile of 100-lb. conductor loop, treated with either of the inductances above mentioned, does not differ materially from 0·0160, while the measured constant is 0·0172 ; thus there is a difference of about 8 per cent. between the calculated and observed values. This is not remarkable, considering that, although the average spacing was 2·44 miles, the actual distance between one coil and another varied from 2·05 to 2·89 miles ; a difference of about 40 per cent. Whenever coils are very irregularly spaced in a loop some loss results from reflections between them. Wonder may be expressed at this irregularity in spacing, and in explanation it may be remarked that at the points selected for the insertion of the coils manholes, jointing pits and other facilities were in existence. It was not considered economical to construct new loading points



A, Cable unloaded ; B, cable loaded with air-core coils ; C, cable loaded with iron-core coils.

FIG 16.—CABLE LOADED AT 1·1-MILE POINTS.

by cutting the cable, and providing new pits for the accommodation of the coils, in order to obtain the slight increase in efficiency which would have resulted from uniform spacing.

The number of coils per wave at 2,000 P.P.S. will be found to be about 5 in the circuits loaded with coils D, and 4·7 in those loaded with coils E. The articulation of speech received over each type of loop was equally good.

At a time when little was known, experimentally, of the extent to which a loop might be improved by the use of loading coils, a long series of tests were arranged upon a cable 38·5 miles in length with an intermediate office 19·4 miles from the end at which the observations were made. Return loops of 2 × 38·5 = 77, or of 2 × 19·4 = 38·8 miles could therefore be arranged, and by joining several such loops in series considerable lengths of circuit were obtained. The cable consisted of 100-lb. per mile conductor loops, the wires being paper-insulated and enclosed in a lead sheath. There were 28 pairs altogether. The electrical constants of the loops before loading were—

Conductor resistance per mile of loop ...	18 ohms
Electrostatic capacity	0·055 mfd.
Inductance...	0·001 henry
Leakance	Negligible

(The capacity of part of the cable was 0·05, and of the remainder 0·06 mfd. per mile. The mean value of 0·055 was obtained by measurement on the whole 38·5 miles.)

Calculated from the usual formula for unloaded subterranean lines, the attenuation constant β = 0·042 per mile. The measured standard cable equivalents of different lengths of circuit are given in column 2 of Table IV, and shown by curve A in fig. 16. The untreated loops are 2·27 times better than standard cable; the attenuation constant per mile is therefore 0·103/2·27 = 0·045 nearly, which result differs from that 0·042 obtained by calculation to the extent of 7 per cent. The cause of this discrepancy was not investigated, as, at the time the trials were conducted, it was considered satisfactory if the observed and calculated results agreed within such a percentage.

A few years prior to the date of the experiments next to be described, the cable had been loaded with air-core coils. The importance of regular spacing was not then fully appreciated, and the loading points were very irregularly distributed along the route, the distances between coils varying as much as 60 per cent. The mean spacing distance works out at 1·1 mile. These old coils had to be removed because the wooden cases, originally employed, failed to keep out moisture, and the insulation resistance of the enclosed coils became very low. Whether the more modern double wooden boxes, already described, will prove more effective has yet to be determined; but it is very doubtful whether any wooden case will last long in an underground pit liable to flooding, and which in any circumstances alternates in condition between "wet" and "dry." The reasons why air-core coils cannot be encased in any metal have been stated already. After the old coils had been removed from the circuits, and the cable wires raised in insulation resistance by desiccation, the previously given trials were made on the unloaded loops.

In the more recent loading of this cable some of the loops were fitted with air-core, and the remainder with iron-core coils; the primary object being to determine which type of

TABLE IV.

Mileage of loop under test.	Standard cable equivalent in miles.		
	Circuit unloaded.	Circuit loaded at 1·1-mile points.	
		Air-core coil F.	Iron ~~Air~~-core coil 507.
38·8	17	8·5	8·0
77·0	34	14·2	12·6
115·8	51	20·0	17·9
154·0	—	25·3	22·2

coil was the better as regards the *clearness* of speech transmitted—*i.e.*, which was the better from the "articulation" point of view. A secondary object—but not less important—was to ascertain what the most economical loading would be, having regard to both the clearness and the loudness of speech received. Incidentally, other interesting points were investigated, such as the measurements of mixed circuits, terminal loss, &c., all of which will be dealt with in due course.

Confining attention to the main points, it may be explained that the two types of coil used in the comparison tests were—

(*a*) AIR-CORE COILS. TYPE F.

Effective resistance ... ... 15·4 ohms at 750 P.P.S.
Inductance ... ... 0·100 henry at 750 P.P.S.

(*b*) IRON-CORE COILS, W.E. CO.'S TYPE 507.

Effective resistance ... 5·4 ohms at 750 P.P.S.
Inductance ... ... 0·133 henry at 750 P.P.S.

It would occupy much too long a time, and too great a space, to consider all the results obtained in the same detailed manner as that hitherto adopted; it is therefore proposed to deal briefly with each experiment, and to give a summary of the deductions after each table of results. Only bear in mind throughout (1) that for air-core coil loaded loops the attenuation constant per mile is calculated from—

$$\beta = \frac{R + 50\,L}{2}\sqrt{\frac{K}{L}},$$

and for those with iron-core coils from—

$$\beta = \frac{R + 80\,L}{2}\sqrt{\frac{K}{L}};\ \text{and—}$$

(2) that the decimal points shown in connection with the measurement of standard cable equivalents were not due to estimation during the trials, but were the result of the previous calibration of the particular standard cables employed in the tests. The steps by which the standard cable length could be increased, varied somewhat as follows:—1·1, 2·1, 3·2, 4·3, &c.

Experiment (*a*). Coils were inserted in the loops at 1·1 mile (average) intervals; some of the circuits being loaded entirely with the air-core, and others with the iron-core type. The measured standard cable equivalents of various lengths of each kind of loaded loop are given in Table IV.

The results are shown by the curves in fig. 16. Reading from the curves, the terminal loss and observed attenuation are found to be as shown below:—

Air-Core Coil Loaded Loops.

Terminal loss = 3 M.S.C.

Excluding terminal loss, the loops are 6·9 times better than standard cable. Observed attenuation constant is, therefore, 0·103/6·9 = 0·0149 nearly per mile.

Calculated β = 0·0141 per mile.

Coils per wave-length at 2,000 P.P.S. = 6·4.

Iron-Core Coil Loaded Loops.

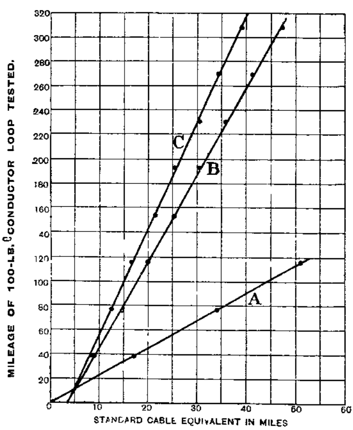
Terminal loss = 3 M.S.C.

Excluding terminal loss, the loops are 8 times as good as standard cable. Observed attenuation constant is, therefore 0·103/8 = 0·0130 nearly per mile.

Calculated $\beta = 0{\cdot}0110$ per mile.
Coils per wave-length at 2,000 P.P.S. $= 5{\cdot}6$.

The quality of speech transmitted over the two types of circuit was very good. A direct comparison was made by placing an air-core coil loaded loop in the position occupied by the standard cable shown in fig. 13, and an equivalent iron-core coil loaded circuit where the "loop under test" is shown in the same figure. An instantaneous change-over from one circuit to the other, by means of the switch, showed that no difference existed between the types of loop so far as the clearness of transmitted speech was concerned. This observation indicates that the presence of iron cores in loading coils does not adversely affect the articulation of the speech received.

Experiment (*b*).—The same coils as were used in experiment (*a*) were next re-arranged so as to be at an average distance apart of 2·1 miles. Again, the actual distance between the coils in each loop varied about 60 per cent.



A, Cable unloaded; B, Loaded with air-core coils; C, Loaded with iron-core coils.

FIG. 17.—CABLE LOADED AT 2 1-MILE POINTS.

Table V shows the results obtained upon different lengths of each type of loaded circuit, and from these results curves B and C in fig. 17 have been drawn.

TABLE V.

Mileage of loop under test.	Standard cable equivalent in miles. Circuits loaded at 2·1-mile points	
	Air-core coil F.	Iron-core coil 507.
38·8	9·0	8·0
77·0	14·7	12·6
115·8	20·1	16·3
154·0	25·3	21·6
192·8	30 6	25 3
231·0	35·8	30 6
269·8	41·2	34·3
308·0	47·5	39·1

From the curves, the terminal loss and observed attenuation appear to be as follows :—

Air-Core Coil Loaded Loops.

Terminal loss = 3·7 M.S.C.

Excluding terminal loss, the loops are seven times better than standard cable. Observed attenuation constant = 0·103/7 = 0·0147 per mile.

Calculated β = 0·0147 per mile.

Coils per wave-length at 2,000 P.P.S. = 4·6.

Iron-Core Coil Loaded Loops.

Terminal loss = 3·8 M.S.C.

Excluding terminal loss, the loops are 8·7 times better than standard cable. Observed attenuation constant = 0·103/8·7 = 0·0118 per mile.

Calculated β = 0·0120 per mile.

Coils per wave-length at 2,000 P.P.S. = 4.

The quality of speech received over each type of loaded circuit was the same and very good.

Experiment (c).—The coils were redistributed in the loops so as to be at an average distance apart of 3·2 miles. In this case the actual variation in the spacing was 31 per cent. The following Table VI gives the standard cable equivalent values of the different loops as measured.

These results are shown graphically by the curves in fig. 18, and the conclusions, so far as observed results are concerned, are :—

Air-Core Coil Loaded Loops.

Terminal loss = 5·4 M.S.C.

Excluding terminal loss, the loops are 6·9 times better than standard cable. Observed attenuation constant = 0·103/6·9 = 0·0149 per mile.

Calculated β = 0·0160 per mile.

Coils per wave-length, at 2,000 P.P.S. = 3·7.

TABLE VI.

Mileage of loop under test.	Standard cable equivalent in miles. Circuits loaded at 3·2-mile points.	
	Air-core coil F.	Iron-core coil 507.
38·8	9·0	8·8
77·0	16·5	14·8
115·8	22·0	19·8
154·0	27·5	23·1
192·8	33·0	28·6
231·0	38·4	31·9
269·8	43·6	36·3
308·0	49·9	41·5

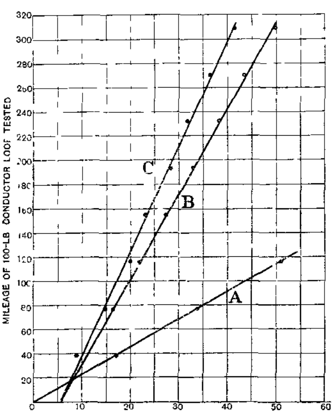


FIG. 18.—CABLE LOADED AT 3·2-MILE POINTS.

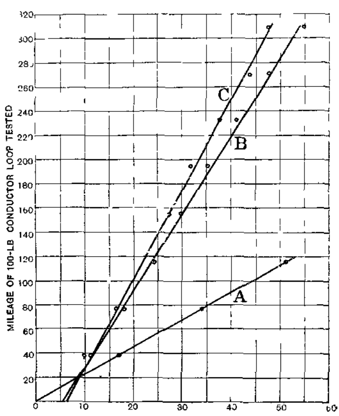


FIG. 19.—CABLE LOADED AT 4·3-MILE POINTS.

Iron-Core Coil Loaded Loops.

Terminal loss = 5·8 M.S.C.

Excluding terminal loss, the loops are 8·6 times better than standard cable. Observed attenuation constant = 0·103/8·6 = 0·0120 per mile.

Calculated β = 0·0131 per mile.

Coils per wave length at 2,000 P.P.S. = 3 3.

The speech transmitted over similar lengths of each type of loaded circuit was equally clear, and the articulation satisfactory.

Experiment (*d*).—Finally, the coils were inserted in the loops at a mean distance of 4·3 miles apart. The actual spacing between the inductances varied by 14 per cent. Table VII shows the measured standard cable equivalents of different lengths of each type of loaded loop.

Curves constructed from these results are shown in fig. 19, the deductions therefrom, respecting terminal loss and observed attenuation, being as follows :—

Air-Core Coil Loaded Loops.

Terminal loss = 5 6 M.S C.

Excluding terminal loss, the loops are 6·2 times better than standard cable. Observed attenuation constant = 0·103/6·2 = 0·0166 per mile.

Calculated β = 0·0172 per mile.

Coils per wave-length at 2,000 P.P.S. = 3·2.

TABLE VII.

Mileage of loop under test.	Standard cable equivalent in miles. Circuits loaded at 4·3-mile points.	
	Air-core coil F.	Iron core coil 507.
38 8	9·8	11·0
77·0	17·9	16·8
115·8	24·2	23·9
154·0	30·0	27·5
192·8	35 2	31·9
231·0	41·2	38 0
269·8	47·5	43·7
308·0	54·8	47·5

Iron-Core Coil Loaded Loops.

Terminal loss = 6·6 M S.C.

Excluding terminal loss, the loops are 7·5 times better than standard cable. Observed attenuation constant = 0·103/7·5 = 0·0137 per mile.

Calculated β = 0·0143 per mile.

Coils per wave-length at 2,000 P.P.S. = 2·8.

In this case the speech received over either type of loaded circuit was of a very inferior quality—too bad, indeed, for commercial purposes.

Although the spacing of the coils in each of the four experiments just described was irregular, the distance variation between air-core and iron-core coils in the respective loops was the same. A very general comparison of results, therefore, may be permitted.

The attenuation constants per mile observed in the abovementioned experiments (*a*) to (*d*), together with the corresponding calculated values, are summarised in the following Table VIII :—

TABLE VIII.

Average distance between coils.	Attenuation constant per mile.			
	Air-core coils F.		Iron-core coils 507.	
Miles.	Observed.	Calculated.	Observed.	Calculated.
1·1	0·0149	0·0141	0·0130	0·0110
2·1	0·0147	0·0147	0·0118	0·0120
3·2	0·0149	0·0160	0·0120	0·0131
4·3	0·0166	0·0172	0·0137	0·0143

From this table and the results and curves (figs. 16 to 19) previously given, it will be seen that with any given spacing the attenuation constant per mile was less for the iron-core than for the air-core coil loaded loops; or, in other words, the "loudness of speech" received over a given length of the former was greater than that received over an equal length of the latter type of circuit. This result is quite in order, as the iron-core possessed a higher inductance, and offered a lower effective resistance than the air-core coils.

Since coils constructed with solid cores of properly selected magnetic material (so-called iron cores) introduce less resistance into a circuit for a given inductance than do air-core coils, and at the same time transmit speech equally clearly, they are the better type of loading coil. To remove doubt upon this point, compare experiments (*b*) and (*c*). Air-core coils F at 2·1, and iron-core coils 507 at 3·2-mile intervals, make the inductance per mile of loop 0 049 and 0·042 henry respectively; from which it would appear that coils F spaced 2·1 miles apart should produce slightly better results than type 507 at 3·2-mile intervals. The reverse effect was found in practice, as will be gathered from the results already tabulated.

The observed and calculated results summarised in Table VIII agree within about 9 per cent., except in the case of the 1·1-mile spacing with iron-core coils, where the cable was, no doubt, overloaded. Leaving the 4·3-mile loading out of consideration because of the inferior quality of speech transmitted, and omitting the 1·1-mile spacing for the reason that the loading was too heavy, attention may be confined to the 2·1 and 3·2-mile treatment. As the attenuation constant per mile of unloaded loop was 0·043, it will be seen that the air-core coils produced an increase of about 300, and the iron-core of approximately 370 per cent. in the volume of speech transmitted over equal lengths of circuit. The formulæ employed in the calculations are, therefore, fairly accurate, since they enable the telephone engineer to foretell an improvement of 370 per cent. correctly within 9 per cent.; and this in the face of very uneven distribution of coils. In passing, it may be noticed that when the coils of either type were close together, the calculated were better than the observed results; but, as the coils were placed further apart, the observed became better than the calculated attenuation constants. This change-over may be due to the variation between too heavy and too light loading. Taking into account the types of coil available, the cost and the improvement produced, the 3·2-mile spacing is the best in the cable under consideration.

As regards quality of speech transmitted, it will be noted that, in each separate trial (*a*) to (*d*), the iron-core coils proved to be as efficient as those of the air-core form. It has been emphasised that the minimum spacing of coils is such that there will be rather more than π coils per wave-length at 2,000 p.p.s., and that this is so seems to be confirmed by the observations which, for convenience, have been extracted, and are given in Table IX :—

TABLE IX.

Average distance between coils. Miles.	Coils per wave at 2,000 P.P.S.		Grade of articulation.
	Air-core.	Iron-core.	
1·1	6·4	5·6	Very good.
2·1	4·6	4·0	" "
3·2	3·7	3·3	Good.
4·3	3·2	2·8	Bad.

The irregular spacing, no doubt, made the 4·3-mile loading a little worse than it might otherwise have proved from the "quality of speech" point of view; and in considering these results it should be remembered that the clearness of transmission was judged mainly by comparison with that obtained through equivalent lengths of standard cable. In one other experiment in connection with articulation, the loading coils were so selected and spaced in the loops that all circuits were practically equal in inductance per mile. The results are summarised in Table X. It is scarcely necessary to mention that any one loop was loaded with only one type of coil.

TABLE X.

Coil spacing in miles.	Inductance per mile of circuit including coils. Henries.	Wave length in miles.	No. of coils per wave at 2,000 p.p.s.	Grade of articulation.
2·1	0·034	11·6	5·5	Very good.
3·2	0·032	11·9	3·7	Good.
4·3	0·032	11·9	2·8	Bad.

NOTE.—Electrostatic capacity = 0·055 m.f. per mile of loop.

The terminal losses observed are shown in Table XI :—

TABLE XI.

Average distance between coils. Miles.	Terminal loss expressed in miles of standard cable.	
	Air-core coils F.	Iron-core coils 507.
1·1	3·0	3·0
2·1	3·7	3·8
3·2	5·4	5·8
4·3	5·6	6·6

The same apparatus was used throughout the trials, but it is interesting to notice that (*a*) the terminal loss with the iron-core was generally greater than with the air-core coils. This is in accordance with the commonly-accepted rule that the greater the inductance placed in circuit, the greater is the terminal loss in apparatus designed for use on unloaded lines ; (*b*) the loss became *greater* with each type of coil as the inductance in the circuit was made *less* by increasing the distance between the loading points. These results are not in agreement with those obtained in the tests upon the 40-lb. conductor cable given in Table II and fig. 15 ; but until more experience has been gained in connection with cables differently loaded, it is not proposed to attempt an explanation of the peculiar variations of "terminal loss."

The loss at the ends of a loaded circuit may be reduced in several ways, typical methods being shown diagrammatically in fig. 20. That denoted by (*a*) is known as "tapering." The central and major portion of the loop is, say, loaded with coils of 100 M.H. inductance, but at each end of the circuit the inductance per coil is reduced, somewhat as shown in the figure. A decrease in terminal loss takes place, but, owing to the fact that the inductance per mile of loop is reduced at the ends, the improvement obtained by loading is not really so great as when all the coils are equally high in inductance. The one effect more or less counterbalances the other, and, so far as present experience shows, this method is not likely to be used in future. In the arrangement marked (*b*) the whole loop proper is loaded with the same type of coil, but is extended at each end by a combination of resistance and capacity virtually comprising an artificial unloaded cable. The extent to which terminal loss is removed by this means depends upon the values of the added

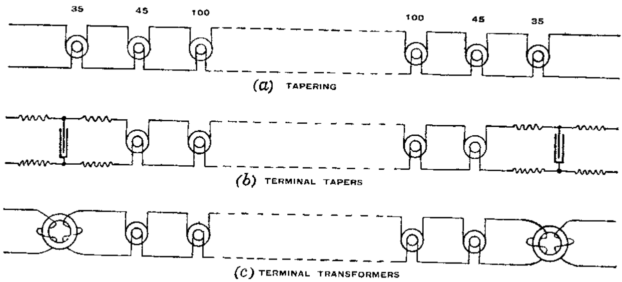


FIG 20.

resistances and capacities; if they be sufficiently high, all the terminal loss may be removed from the loaded line. No great benefit results, however, for the amount of artificial cable required to remove all the terminal loss has as great a standard cable equivalent as that of the terminal loss itself. Most loaded lines in this country are subject to extension by actual unloaded loops, and terminal loss automatically disappears, but this point is mentioned again a little later on. The method shown by (*c*) in the figure consists of terminating the loaded loop through one coil of a specially designed transformer at each end, the telephone set being joined to the other transformer winding. Such apparatus are capable of appreciably reducing terminal loss, but if not properly designed they introduce as much loss in transformation as they remove from the termination of the loaded circuit, and no net gain results. They may prove of value in cases where telephones have to be fitted direct to the ends of loaded loops, or upon junction lines subject to extension by short subscribers' lines.

There appears to be no necessity for the provision of any special device for reducing terminal loss on lengthy loaded *trunks* at present, for the reason that such circuits are normally subject to extension by other trunks, junction and subscribers' lines, and these are generally, but not always, sufficiently long to eliminate the end loss. The details of one experiment upon terminal loss removal by unloaded extensions are given in Table XII :—

TABLE XII.

Length of loaded loop, miles.	Standard cable equivalent of extensions each end, M.S.C.	Terminal loss, M.S.C.	Type of coil.
77	Nil.	5·4	Air core.
,,	2·0	2·7	,, ,,
,,	4·0	0·9	,, ,,
,,	5·5	Nil.	,, ,,
77	Nil	5·8	Iron core.
,,	2·0	3·7	,, ,,
,,	4·0	1·9	,, ,,
,,	5 5	1·0	, ,,
,	7·5	Nil.	, ,,

Many experiments have been made upon mixed circuits, *i e.*, those formed by joining lengths of unloaded aerial or underground line in series with loaded underground loops. So far as measurement by the standard cable method shows, there is no loss due to joining loaded and unloaded lines of any kind in series in any order. As typical cases, consider the following three loops :—

(*a*) An aerial loop measuring 12·6 M.S.C.

(*b*) An underground unloaded loop equivalent to 7·5 M.S.C.

(*c*) A loaded underground loop which, with terminal losses, was equal to 14·8 M.S.C.

The sum of these measurements is 12·6 + 7 5 + 14·8 = 34·9 M.S.C.

The terminal loss on the loaded loop was 5·8 M.S.C.

First, the three lengths were joined in series in the order (*a*), (*c*), (*b*)—*i.e.*, the loaded loop in the middle. The observed standard cable equivalent from either end (*a*) or (*b*) was 29 miles, or a reduction of 5·9 M.S.C. on the sum of the equivalents of the parts. This merely means that the terminal loss on the loaded length was wiped out, and 29 M S C. represents the true equivalent of the combination. Secondly, the same lengths were arranged in the order (*a*), (*b*), (*c*), *i.e.*, the loaded loop at one end. The measured equivalent from the aerial end (*a*) was 29 M.S.C. as before, but that from the loaded end (*c*) was 31·9 M.S.C. The latter value is high because of the terminal loss at the end of (*c*).

In all of the hundreds of cases taken, it has been found that, *excluding terminal loss on loaded lines*, the standard cable equivalent of any number of different loops joined in series in any order is equal to the sum of the equivalents of the parts measured separately. This is of some importance, but particularly in connection with the measurement of the standard cable equivalents of differently designed unloaded loops between the same towns, for, letting three such loops be A, B and C respectively, suppose—

$$A + B = 15 \text{ M.S.C.}$$
$$B + C = 19 \quad ,,$$
$$A + C = 12 \quad ,,$$

$$\text{then } 2\,(A + B + C) = 46 \quad ,,$$

$A + B + C = 23$ M.S.C., and by subtracting the observed values, the equivalent of each loop may be found.

Before leaving the subject of underground loading by coils, it may be of interest to note one effort which was made to determine what ill effect resulted from the irregular spacing of the inductances in the trials, results of which were given in Table VI and fig. 18. Two special loops were arranged, one to include the most irregular portions of the loading, the other to embrace the most regular. For equal lengths of circuit containing the same number of coils the standard cable equivalents were found to be 25·8 and 24·2 miles respectively. There was thus a loss of about 6·5 per cent. in transmission efficiency, due to the greater irregularity of the loading in the one loop as compared with that in the other. When it is considered that the more irregular of the two circuits thus made up did not differ in its want of symmetry from the more evenly loaded loop by an amount equal to that by which the normal spacing differs from perfect regularity, it will be agreed that 10 per cent. is not too high an estimate of the loss of efficiency due to the normal irregular spacing. It is important that the loading points should be spaced regularly, at least within 5 per cent.

A coil may be regarded as loading a length of the loop equal to half the normal spacing on each side of it. From this point of view, the spacing of inductance coils should be such that the first and last are at a distance from the cable ends equal to half that between any other two coils. This procedure has the advantage that, should two similarly loaded loops be joined in series, the spacing throughout is kept uniform. It also ensures that the terminal loss shall not be abnormal, for it is found that the nearer the first and last coils are to the ends of the circuit, the greater is such loss. The half-normal spacing lengths at each end, in fact, act as small terminal tapers when speaking takes place over the loaded loop only.

Conductors Insulated with Gutta-percha.—Some writers have expressed doubt as to the applicability of the ordinary formulæ for loaded paper-insulated cable circuits to loops insulated with gutta-percha. With the object of determining whether such doubt was justified, a practical trial was arranged upon loops, each conductor of which weighed 40 lb. per mile nominally, had an average overall diameter of 50 mils, and was covered with gutta-percha to a mean overall diameter of 176 mils. The electrical constants of these unloaded loops were :—

Conductor resistance per mile of loop ...	44 ohms.
Electrostatic capacity	0·001 henry.
Inductance	0·13 mfd.

A total length of 105 miles of loop, made up of seven 15-mile sections, was arranged, and each 15-mile section measured as nearly as possible 16 miles of standard cable. The attenuation constant per mile of unloaded loop was, therefore :—

$$0{\cdot}103 \times 16/15 = 0{\cdot}11.$$

By calculation from the ordinary formula for unloaded lines the same value is obtained.

In loading these loops, air-core coils having an inductance of 0·083 henry, and offering an effective resistance of 13·4 ohms at 750 p.p.s., were placed in circuit at 1·5-mile intervals. The gutta-percha wire was placed in tanks and covered with water, but the coils were kept dry, because the coil-cases were not watertight. The standard cable equivalents observed after loading were as follows :—

15 miles of loaded loop	... ...	13·0 M.S.C.
30 " "	... ...	19·0 "
45 " "	... ...	25·3 "
60 " "	... ...	31·6 "
75 " "	... ...	37·4 "
90 " "	... ...	43·3 "
105 " "	... ...	49·6 "

These results are shown graphically in fig. 21.

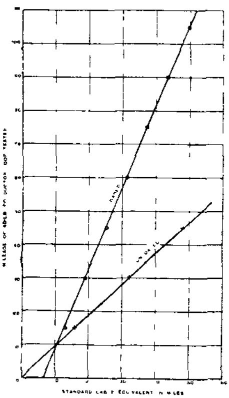


FIG. 21.

From the preceding figures, or the curve, it will be found that each 15 miles of loaded loop is as nearly as possible equal to 6·1 miles of standard cable, so that the observed attenuation constant per mile is 6·1 × 0·103/15 = 0·042 nearly. By calculation, using the formula for air-core coils, the value β = 0·043, nearly, is obtained. It therefore appears that the formulæ are applicable whether the insulating material be paper or gutta-percha, but it should be noted that these results were obtained with the gutta-percha wire submerged. When the water was drained off, and the wire allowed to dry, very variable results were obtained, no doubt due to the well-known facts that the effective insulation resistance of gutta-percha-covered wire, when dry, is itself variable, and lower in value than when immersed in water. Another interesting point in connection with these experiments is that 45 miles of the loop used had an insulation resistance of about 1 megohm per mile, while a second equal length had an insulation resistance of over 500 megohms per mile. A direct comparison between these lengths did not reveal any difference in telephone transmission efficiency.

Continuous Loading.—This, as previously mentioned, is generally effected by covering each conductor of the loop with one or more layers of finely-drawn iron wire. So far as is known, no practical experience of this type of loading actual circuits has yet been obtained in this country, but laboratory tests have been made upon short lengths. In one experiment three equally long lengths were made up in the following manner:—

1. Each conductor of solid copper, 79 mils in diameter, and weighing 100 lb. per mile. Wires merely insulated with paper to a given diameter.

2. Each conductor similar to those in (1), but covered with three closely-wound layers of No. 16 S.W.G. annealed iron wire, and then insulated with paper to the same dimensions as the loop mentioned in (1).

3. Each conductor consisted of a strand of seven copper wires, the strand weighing 100 lb. per mile, covered with three closely-wound layers of iron wire, and insulated as in the preceding case (2).

The general result of the experiment was to show (*a*) that at 1,000 p.p.s. the effective resistance of the iron-covered solid copper was 90 per cent., and of the iron-covered strand 64 per cent. greater than that of the untreated loop; (*b*) that the electrostatic capacity of the iron wound loops was roughly twice as great as that of the untreated loop; (*c*) that the inductance per mile of the iron-covered loops was 0·0414 henry, while that of the untreated loop was only 0·0012.

The increase in the resistance with frequency is serious, but, of course, the capacity might be reduced by increasing the overall diameter of the insulation. In all probability the iron wire used in the experiments was somewhat too thick, so that no great notice will be taken of the results obtained. Calculation showed that the attenuation constant per mile of the iron-wound solid copper conductor loop was half that of the untreated circuit, *i.e.*, the iron wire produced an improvement of 100 per cent., and its use may, therefore, be considered in any case where the fitting of inductance coils is impossible. It is not believed that the present known methods of continuous loading can ever be made as efficient as coil loading, for the reason that so much more iron is required to obtain a given increase in inductance in the loop, and the iron is not used to its best advantage. Finally, the cost of iron-covering such wires is high, and they take up much more space than ordinary loops.

Some details of continuously loaded cables laid abroad have appeared in various periodicals, but from what can be deduced from the figures given, it would appear that much of the efficiency claimed results from low conductor resistance and capacity, and not from the presence of the iron windings over the conductor.* It is, however, hoped that further information will be available shortly, when, perhaps, a further paper may be contributed on the subject.

* Since this paper was read, telephone transmission tests have been made on actual continuously loaded loops of considerable length; the improvement due to the iron was found to be 60 per cent. for large conductors, but no doubt 100 per cent. increase in speaking distance might be obtained upon loops made up of smaller copper wires.

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