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INSTITUTION OF
POST OFFICE ELECTRICAL ENGINEERS.

The Loading of Aerial Lines and their Electrical Constants.

BY

J. G. HILL.

A PAPER

Read before the London Centre of the Institution on 12th January, 1914.

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THE LOADING OF AERIAL LINES AND THEIR ELECTRICAL CONSTANTS.

The literature relating to Telephone Transmission has grown rapidly in recent years, and there is now ample information available for those who can follow the somewhat complex mathematics connected with the subject. Notwithstanding the general information obtainable, however, the application of the laws of Transmission to particular cases often renders special investigation and experiments necessary, before a reliable conclusion can be reached. This remark applies particularly to the loading of aerial lines in this country. In considering the transmission efficiency of any circuit, three principal things should be taken into account:—

(1) The attenuation constant of the circuit.

(2) The characteristic Impedance of the circuit (*i.e.*, the Impedance of a "long" line).

(3) The quality of the transmission in the circuit, *i.e.*, how far it departs from the distortionless condition.

The laws regulating the loading of underground circuits are now well understood in this country, and from the general point of view, the loading problem is the same for overhead as for underground lines, but there are important differences of degree in these cases, and the differences are such as to require experimental investigation. As regards (1) above, the efficiency of an aerial loaded line is theoretically liable to much greater fluctuations than that of an underground line, owing to the much greater variation of leakance in the former than in the latter. The amount of this variation at alternating current frequencies corresponding to those of the human voice, called in this case for special investigation. As regards (2) the Impedance of a loaded aerial line is much greater than that of an underground line, and the effects of reflection are correspondingly serious, and repay careful investigation. Finally, the quality of speech in an underground circuit is usually improved by loading, but, from theoretical considerations, this could not be expected in aerial lines, and observations were necessary to ascertain whether actual deterioration of quality occurred, and, if so, to what extent.

The problems of transmission are so complex and cover so large a field, that notwithstanding all that has been written, there may, I hope, be some advantage in presenting the subject, as applied to loading, from a point of view which appeals to the writer as having some advantage on the score of simplicity and clearness. With this object in view, therefore, I make no apology for re-stating facts which are not new.

The most essential things for the practical Engineer to know in order to deal with the application of loading to working conditions are indicated under (1) (2) and (3) above, and I propose to attempt to throw some light on these points as simply as possible before dealing with the experimental part of the subject. Mathematical expressions for the first two quantities are dealt with in the following pages and are more fully developed in appendix No. 1. The considerations involved in (3) are dealt with on page 15 under the heading " Some remarks on the distance apart at which loading coils should be spaced."

It will, perhaps, be of interest to first define loading. In its perfect form, loading is the addition of inductance to a telephone circuit in such a way that the circuit acts at all frequencies, and at any instant, as if the current in it were a direct current without any frequency effects. This does not, of course, mean that the current in the circuit will not be subject to attenuation, for in any leaky circuit, direct currents, as well as alternating currents, are attenuated Ideal loading simply corrects the distortion, and the additional attenuation effects, which are due to frequency. Such a circuit is said to be distortionless, because the attenuation of signals is the same at all frequencies. The study of this special case is instructive from the practical point of view, although perfect loading is not attainable, and I therefore propose to review the principal facts in connection with it.

DISCUSSION OF THE DISTORTIONLESS CIRCUIT AND THE MINIMUM ATTENUATION CONSTANT.

As shewn in Appendix I, the attenuation constant is

$$\beta = \sqrt{\tfrac{1}{2}\sqrt{(R^2 + p^2L^2)(S^2 + p^2K^2)} + \tfrac{1}{2}(RS - p^2KL)} \quad \ldots\ldots(1)$$

and this applies to any type of telephone circuit, whether it be loaded or unloaded, overhead, or underground.

Now, if a loaded circuit is to act as though only direct currents were concerned, the effects of frequency must disappear, that is, the frequency factor becomes 0. Equation (1) then becomes

$$\beta = \sqrt{\tfrac{1}{2}\sqrt{(R^2 + 0)(S^2 + 0)} + \tfrac{1}{2}(RS - 0)} \text{ that is}$$

$$\beta = \sqrt{\tfrac{1}{2}\sqrt{R^2S^2} + \tfrac{1}{2}RS} = \sqrt{RS} \quad \ldots\ldots(2)$$

and this is the least possible β for the given R and S. (See Appendix 3).

Equation (2) has been obtained on the assumption that the frequency of the current is nil; but it is easy to show that if the circuit constants bear the relation LS = KR to each other, we obtain the same equation for β in the case of alternating currents at any frequency, *i.e.*, we obtain the conditions necessary for a distortionless circuit with the minimum β. This is proved in Appendix 3 on the assumption that LS = KR.

This latter relation may easily be proved as follows:—

The characteristic impedance of any electrically long telephone circuit is

$$\sqrt{\frac{R + jpL}{S + jpK}} = Z_0 \quad \ldots\ldots(3)$$

(See Appendix I).

If we make the same assumption as before, *i.e.*, that $p = 0$ we get

$$Z_0 = \sqrt{\frac{R + 0}{S + 0}} = \sqrt{\frac{R}{S}} \quad \ldots\ldots(4)$$

i.e., the characteristic impedance of a distortionless circuit *acts* as though it were $\sqrt{\frac{R}{S}}$. But the characteristic impedance is in any case really $Z_0 = \sqrt{\frac{R + jpL}{S + jpK}}$

$\therefore$ in the case supposed $\sqrt{\frac{R + jpL}{S + jpK}} = \sqrt{\frac{R}{S}}$ therefore

$\frac{R + jpL}{S + jpK} = \frac{R}{S}$ or $RS + jpLS = RS + jpKR$, that is

$jpLS = jpKR$

$$\text{and } LS = KR \text{ (at any frequency)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

From formulas Nos. (2) and (5) an expression (No. 9) is derived which is of great use, as it immediately leads to a formula used in practical work.

The steps leadings to the derived formula are as follows:—

$$S = \frac{KR}{L} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

$$\text{and } R = \frac{LS}{K} \text{ from (5)} \quad \ldots\ldots\ldots\ldots\ldots\ldots(7)$$

Next, in equation (2) substitute the value of S from (6) and we get

$$\beta = \sqrt{R\,\frac{KR}{L}} = \sqrt{R^2\,\frac{K}{L}} = R\sqrt{\frac{K}{L}} \ldots(8)$$

$$\text{But } R = \frac{LS}{K} \text{ from (7)}$$

$$\therefore\ 2R = \frac{LS}{K} + R$$

$$\text{and } R = \frac{R + \frac{S}{K}L}{2}$$

Substitute this value of R in (8) and we get

$$\beta = \left(\frac{R + \frac{S}{K} L}{2} \right) \sqrt{\frac{K}{L}} \dots\dots\dots\dots\dots\dots (9)$$

i.e., in any circuit whatever having the constants R, S, L and K we have, by formula (9) the distortionless condition when $R = \frac{S}{K} L$, that is we have the same β for all frequencies, on the assumption that the constants themselves do not alter with frequency. Now, the equality of R with $\frac{SL}{K}$ is, as a rule, not realised in unloaded circuits. Generally speaking, $\frac{S}{K}$ L is much less than R. The factor $\frac{S}{K}$ L, however, may in such a case be made equal to R, either by increasing S or L. This is obvious from inspection of (7). If S is increased, however, β is also increased. This is clear from inspection of the formula $\beta = \sqrt{RS}$ for if S be increased to $S + S_1$ where S_1 is the increment added to produce the distortionless condition, we have $\beta = \sqrt{R(S + S_1)}$. If, however, the distortionless condition be produced in the same circuit by increasing L and not S, $\beta = \sqrt{RS}$ (See Appendix No. 3), which is clearly less than $R\sqrt{(S + S_1)}$.

Formulas (2) and (9) indicate a distortionless circuit in the sense that the frequency factor p is eliminated, and to avoid confusion the word distortionless is used in that sense—yet in practice S always increases with frequency and R increases largely in certain cases with frequency. It has been shown that

$$\sqrt{RS} = \frac{R + \frac{S}{K} L}{2} \sqrt{\frac{K}{L}}$$

when LS = KR. With that proviso, therefore, equation (9) gives the minimum β and we can most usefully attain it by varying L or, in other words, by loading. With some modifications now to be explained, this is the formula used for practical work.

A SIMPLE METHOD OF DEDUCING A WELL KNOWN FORMULA FOR THE PRACTICAL MINIMUM β.

It will be useful to examine formula (9) a little more closely.

If we suppose R, S and K to have a fixed value for any circuit, L being variable, the equation is of the form

$$\frac{A + Bx}{2}\sqrt{\frac{C}{x}}$$

A, B and C being constant and x variable.

We have seen that this expression is a minimum value when $A = Bx$. The formula is a general one, it will apply to any quantities whatever, whether electrical or not, where A, B and C are constants and x is a variable. We can now extend the use of the formula. Equation (9), as it stands, is purely theoretical, inasmuch as we cannot increase L (as the equation assumes) without at the same time increasing R. Every loading coil added to the circuit has resistance as well as inductance, but, in a well designed coil, the increase in resistance can be made proportional to the added inductance, so that the ratio $\frac{R_1}{L_1}$ is constant (R_1 is the effective resistance of the coil at a given frequency, and with a specified current flowing through the coil, and L_1 is the inductance in henrys in the coil). The frequency taken in this case is 800 periods per second (generally taken as the frequency which gives the same attenuation as telephonic speech).

Formula (9) now becomes

$$\beta = \frac{R + R_1 + \frac{S}{K} L}{2}\sqrt{\frac{K}{L}}$$

where R_1 is the added resistance of the loading coil. If L_1 be the coil inductance and L the total inductance per unit length, then when the natural inductance is negligible (as in Cable circuits) and we have

$$\beta = \frac{R + \frac{R_1}{L_1} L + \frac{S}{K} L}{2}\sqrt{\frac{K}{L}} \text{ that is}$$

$$\beta = \frac{R + \left(\frac{R_1}{L_1} + \frac{S}{K}\right)L}{2}\sqrt{\frac{K}{L}} \quad \dots\dots\dots(10)$$

But, as stated above, $\frac{R_1}{L_1}$ is a constant at a given frequency, as also $\frac{S}{K}$ is assumed to be, and, therefore, $\frac{R_1}{L_1} + \frac{S}{K}$ is a constant, the equation is, therefore, of the form

$$\frac{A + Bx}{2}\sqrt{\frac{C}{x}}$$

and we have the minimum value of the equation when $Bx = A$, that is when

$$\left(\frac{R_1}{L_1} + \frac{S}{K}\right)L = R, \text{ or } L = \frac{R}{\frac{R_1}{L_1} + \frac{S}{K}} \quad \dots\dots\dots\dots\dots\dots(11)$$

In the case of any circuit we wish to load, therefore, we can always find the inductance which will give the minimum β (βm).

Formula (11), as stated, assumes that the circuit has no natural inductance. In an overhead circuit, however, this is not sufficiently accurate and it is necessary to consider the matter more fully. It can be shewn that the best inductance for the circuit in such a case is

$$L = \frac{R - \frac{R_1}{L_1} L_n}{\frac{R_1}{L_1} + \frac{S}{K}} \quad \text{(as is already known)}$$

where L_n is the natural inductance of the circuit.

Now the effect of $\frac{R_1}{L_1} L_n$ on βm may be neglected, and in that case it is sufficient to use formula (11) and deduct L_n from the value of L given by the equation. The amount remaining after the deduction is the inductance per unit length to be added by loading.

It may be remarked that formulas (9), (10), and (18) hold good theoretically in cases where pL is great in comparison with R, and pK with S whether $R = \left(\frac{R_1}{L_1} + \frac{S}{K}\right) L$ or not. For proof, see Appendix 2 and Mr. A. W. Martin's paper, "The Loading of Underground Circuits," page 20). When equation 11 holds good, however, the β we obtain is the least possible with the particular type of coil used. Formula (9) holds in practice for unloaded aerial copper circuits above 200-lbs. per mile in weight. Formula (18), page 24, holds for all aerial loaded circuits and No. (10) holds for any underground loaded circuits if the conductors are not less than 40-lbs. per mile in weight. Outside these limits the formula may not hold. Formula No. (1), however, holds good in any case.

OBSERVATIONS REGARDING S/K.

If $\frac{S}{K}$ can be regarded as a constant, then calculation is materially simplified. A few remarks on the subject may, therefore, be of use at this stage.

In submarine cable circuits for example, the dielectric, after treatment in the process of manufacture, has a normal leakance S, which can be predicted if the capacity K of the cable and the frequency n at which the measurement is made are known. If either the capacity is varied by altering the thickness of the dielectric, or the frequency is varied, the relation between the quantities is usually taken to be such that $\frac{S}{2\pi n K}$ is a constant, and this is found to be very approximately correct. If, however, S varies independently (say owing to low insulation) the relation does not hold, but if in any case a given capacity, and leakance, at a given frequency are known (as in aerial lines) we can in that limited sense again take S/K as a constant.

EFFECT OF LEAKANCE ON β.

In any series loaded circuit where pL is great in comparison with R, and pK with S, leakance should be avoided where practicable,

even in the case where an infinite amount of inductance would be required to produce the minimum β and an amount of leakance would reduce the amount of inductance required. This is not always understood and I propose to analyse the matter further. Equation (9) may be written

$$\beta = \frac{R}{2}\sqrt{\frac{K}{L}} + \frac{S}{\frac{K}{2}} L \sqrt{\frac{K}{L}}$$

$$\text{that is} = \frac{R}{2}\sqrt{\frac{K}{L}} + \frac{S}{2}\sqrt{\frac{L}{K}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

But if S = 0 and pL is great in proportion to R (as it usually is in loaded circuits) we see from Appendix 2 that

$$\beta = \frac{R}{2}\sqrt{\frac{K}{L}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (12a)$$

that is to say the circuit may be distortionless without leakance, and it may be seen by inspection that (12) is greater than (12a) by the amount of the leakance factor on the right of (12). In practice, the left hand member of (12) and also (12a) takes the form $\frac{R + R_1}{2}\sqrt{\frac{K}{L}}$ but the reasoning given above is not in any way altered by that circumstance if $\frac{R_1}{L^1}$ is constant. The outcome of the matter is that if there be no leakance, formula (12a) gives β in a loaded circuit, but if leakance is present then formula (12) must be used and this is equal to (12a) + the quantity on the right hand side of (12). It is clear, therefore, in such a case that leakance increases attenuation. In practice also an increase in S would mean an increase in K, and this would further increase the unsatisfactory effect produced by an increase of S. Again, leakance is also a principal cause of increase of attenuation with frequency in aerial telephone circuits and is very objectionable on that account. See Fig. (6). (In some unloaded telegraph circuits where distortion is present and harmful, an increase in leakance may, however, improve matters).

PRACTICAL FORMULA FOR MINIMUM β.

The formula here developed is sometimes useful in practical work.

In formula (10), when $R = \left(\frac{R_1}{L_1} + \frac{S}{K}\right) L$ we have

$$\beta = \frac{R + \left(\frac{R_1}{L_1} + \frac{S}{K}\right) L}{2} \sqrt{\frac{K}{L}}$$

$$= \frac{2R}{2} \sqrt{\frac{K}{L}} = R \sqrt{\frac{K}{L}} = \sqrt{R^2 \frac{K}{L}}$$

$$= \sqrt{R \times \left(\frac{R_1}{L_1} + \frac{S}{K}\right) L \times \frac{K}{L}}$$

$$= \sqrt{KR \left(\frac{R_1}{L_1} + \frac{S}{K}\right)}$$

$$= \beta m \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (13)$$

i.e., the minimum β, this equation is already known.

FORMULA SHEWING AMOUNT OF VARIATION FROM MINIMUM β WITH A GIVEN INDUCTANCE.

On account of expense and to avoid high values of impedance, it may not be economical to load a circuit fully, and it therefore becomes important to find how far any given inductance increases the value of β as compared with the minimum β, and for this purpose I have developed the following formula:—

Formula (10), as shewn, gives the β for the greater part of loaded circuits. Dividing (10) by (13), we get

$$\frac{\beta}{\beta m} = \frac{\sqrt{\frac{L}{L_1}} + \sqrt{\frac{L_1}{L}}}{2}$$

where L is now the inductance giving the minimum β and L_1 any inductance selected for loading purposes and giving a different β.

If P_v is the percentage of variation from the minimum β then

$$P_v = 50 \left[\left(\sqrt{\frac{L}{L_1}} + \sqrt{\frac{L_1}{L}}\right) - 2\right] \ldots\ldots\ldots\ldots (14)$$

i.e., with any loading L_1, the attenuation constant will vary from the minimum attenuation constant βm, which is obtained by an amount of Inductance L by a percentage P_v. This percentage is shewn graphically in Fig. 1. Formula (13) is theoretically correct for cables, and requires slight modification in theory for open lines, but, in practice, it is also correct to within less than 2% for all open lines. Although this formula gives the minimum β attainable with given values of $\frac{R_1}{L_1} + \frac{S}{K}$, yet it will readily be seen that there is interest in making these constants as small as possible.

The full formula for minimum β in aerial lines is

$$\beta m = \sqrt{K\left(R - \frac{R_1}{L_1} L_n\right)\left(\frac{R_1}{L_1} + \frac{S}{K}\right)} \ldots\ldots (14a)$$

SOME REMARKS ON THE DISTANCE APART AT WHICH LOADING COILS SHOULD BE SPACED.

The complex attenuation constant

$$a = \sqrt{(R + jpL)(S + jpK)} \ldots\ldots\ldots\ldots\ldots\ldots (15)$$

(See Appendix 1), implies not only that the current changes in magnitude by a given percentage per unit length in an infinite line, but also that the sine wave revolves through a definite angle per unit length.

(15) may also be represented thus:

$$a = \beta + j\alpha$$

The imaginary part of a (that is $j\alpha$) merely states that the wave has revolved α radians in unit length, *i.e.*, the phase at unit distance differs from that at the beginning of the line by that angle. Consequently, at any length l it differs by $\alpha\, l$ radians. When $\alpha\, l = 2\pi$ then the wave has revolved through a complete cycle. The wave length is therefore

$$l = \frac{2\pi}{\alpha} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15a)$$

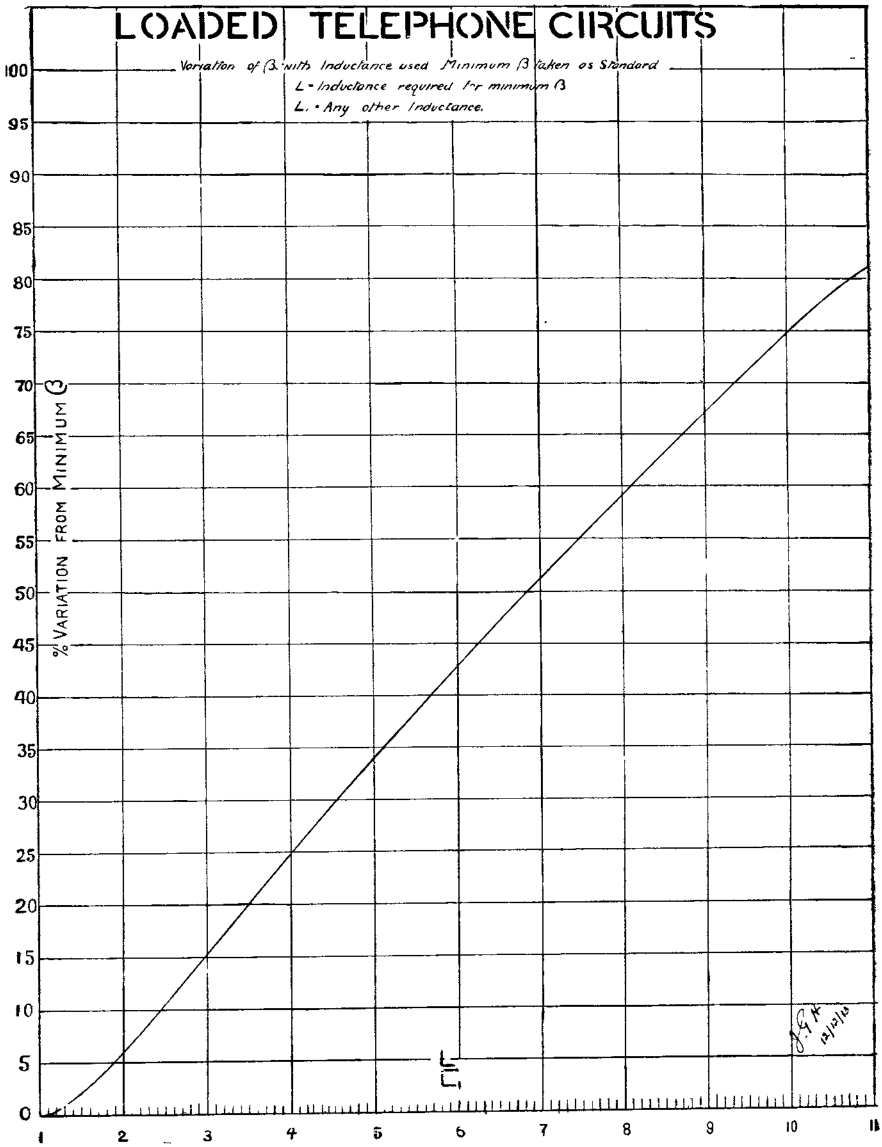
LOADED TELEPHONE CIRCUITS
Variation of β with Inductance used Minimum β taken as Standard
L = Inductance required for minimum β
L₁ = Any other Inductance.
% VARIATION FROM MINIMUM β
100
95
90
85
80
75
70
65
60
55
50
45
40
35
30
25
20
15
10
5
0
L/L₁
1
2
3
4
5
6
7
8
9
10
11

FIG. I.

If we know the frequency (*i.e.*, the number of waves impressed on the line per second), then the velocity (V) of propagation is easily obtained, for the wave length is known and it is passed over in a time $t = \frac{1}{n}$ where n is the number of waves per second. We have $\frac{l}{t} = V$, and, by substitution from (15a)

$$\frac{2\ \pi}{\alpha\ t} = V$$

But $t = \frac{1}{n}$ and $\therefore V = \frac{2 \pi n}{\alpha}$

Now $\alpha = p\sqrt{KL}$

in most loaded circuits (See Appendix 4)

that is $\alpha = 2 \pi n \sqrt{KL}$

$$\therefore V = \frac{2 \pi n}{2 \pi n \sqrt{KL}} = \frac{1}{\sqrt{KL}} \quad \text{.......................(16)}$$

It is obvious from this, that the velocity becomes less as the inductance is increased. The effect of the decreased veloctiy is serious and costly from a loading point of view. It can be inferred theoretically and proved experimentally that as the line becomes more heavily loaded, the coils must be spaced closer and closer* in order to maintain a uniform clearness of speech with different values of loading, and this, of course, means additional expense. It is of interest to compare the rule adopted in different countries.

The rule adopted in England is the following for cable circuits. It is based on experimental results. In the case of overhead lines the clearness of speech is greater than in the case of cables, and probably the figure may be a little exceeded with safety. The rule is

KDL^1 must not exceed 25 ....(17)

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* (See Kennelly's Hyperbolic Functions, pages 148/155; Breisig—Theoretische Telegraphie, page 328; Martin—Loading of Telephone Cables, page 23; Prof. Fleming's—Propagation of Electric Currents in Telephone and Telegraph Conductors, page 130).

where K is the capacity in microfarads per mile,

D the distance apart in miles,

L^1 the inductance of the coil in millihenrys.

In America, the rule is that the advancing wave must strike not less than 7,000 coils per second. [See (16)].

These two rules lead to similar results, although the American rule requires rather more coils and should, of course, give a little clearer speech at a correspondingly greater cost.

The German rule until recently was that the advancing wave shall strike 10,000 coils per second.

I am not quite sure whether this has now been modified.

EFFECT OF LINE IMPEDANCE (INCLUDING CHARACTERISTIC IMPEDANCE).

If a current, I, of a definite amount $\frac{V}{Z_0} = I$ (where V is the impressed voltage and Z_0 the characteristic impedance) be observed at the beginning of a uniform and long line, then, by consideration of the attenuation constant and the circuit length, the current at any other point of the line can be found. It frequently happens, however, in transmission calculations, that the absolute value of the current is not required, but only its relative value as compared with the current in a standard line (the standard cable). So long as the impedances of the lines compared do not differ unduly (and this is largely the case in fairly long unloaded lines) the relative efficiency of the lines may be obtained by comparing their attenuation constants and lengths. This remark, as a general rule, does not apply to loaded lines. In such a case, loading materially increases the impedance and correspondingly reduces the current entering the circuit.

It can be shewn that the best loading for a short line is not the same as that for a long line. Also, if the telephone used is not of suitable impedance (the best result is obtained when the receiving apparatus impedance equals that of the line and has an angle equal and opposite to that of the line)* further losses arise. This

* See Kennelly's "Hyperbolic Functions," Appendix J.

part of the subject is dealt with in Mr. A. G. Lee's Paper on "Telephone Transmission Reflection and its Effects," published in the Abstracts of the I.P.O.E.E., and in Mr. Aldridge's Paper on "Practical Application of Telephone Calculations," read before the Institution of Electrical Engineers last year (1913).

In the investigation dealt with in this paper the losses were investigated experimentally and generally compared with theoretical values.

DETAILS OF EXPERIMENTS.

TESTS TO DETERMINE THE ELECTRICAL CONSTANTS OF UNLOADED AERIAL LINES.

The electrical constants of aerial telephone lines at telephone frequencies are not necessarily those obtained with steady currents. The formula for the attenuation constant is only true when the values of R, L, S and K are effective values, *i.e.*, the values obtained in Alternating Current measurements at some particular frequency. In the special case we are considering the frequency which is generally accepted as giving the same attenuation as the human voice is 800 periods per second. When such measurements are decided upon, it is much easier to obtain the required constants by observations on a relatively short length of unloaded aerial line than on a similar line loaded, but once having obtained the constants they are available for use when such circuits are loaded.

In the case dealt with in this paper, a length of 30 miles of unloaded aerial 300-lbs. copper circuit loop was kept under observation from November, 1912, to February, 1913, and a number of measurements were made from St. Albans Post Office by means of a "Franke" machine (a description of this machine will be found in the "Electrician" of October 24th, 1913). Tests were taken at different frequencies varying between $2\pi n = 3{,}000$ to $2\pi n = 7{,}000$ (n being the number of periods per second) in order to find the amount of attenuation and distortion, as well as the circuit constants. The following table shews how the electrical constants of the circuit as previously accepted from D.C. measurements of the ohmic resistance, calculated values of the mean capacity and inductance of such circuits,

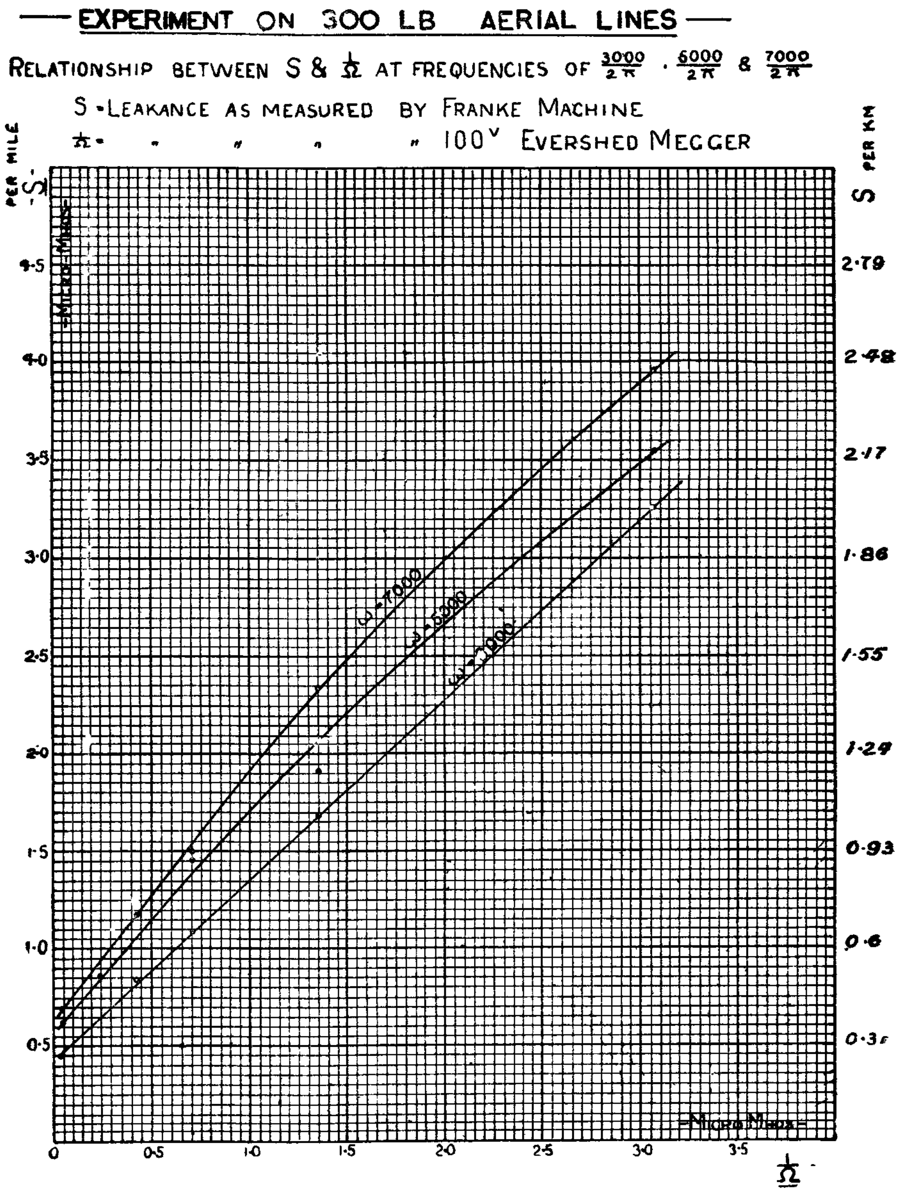

EXPERIMENT ON 300 LB AERIAL LINES
RELATIONSHIP BETWEEN S & $\frac{1}{\Omega}$ AT FREQUENCIES OF $\frac{3000}{2\pi}$. $\frac{6000}{2\pi}$ & $\frac{7000}{2\pi}$
S - LEAKANCE AS MEASURED BY FRANKE MACHINE
$\frac{1}{\Omega}$ - " " " " 100^V EVERSHED MEGGER
PER MILE -S-
-MICRO-MHOS-
S PER KM
ω = 7000
ω = 6000
ω = 3000
-MICRO-MHOS-
0 0·5 1·0 1·5 2·0 2·5 3·0 3·5
0·5 1·0 1·5 2·0 2·5 3·0 3·5 4·0 4·5
0·31 0·6 0·93 1·24 1·55 1·86 2·17 2·48 2·79

FIG. 2.

and an assumed figure for leakance, varied from the mean results found by experiment:—

Values hitherto used in calculation.	Mean values found by experiment at frequency $2\pi n = 5{,}000$ = 800 periods per second, nearly.
R 6.7 ohms	6.75 ohms
L 0.00355 henrys	0.00383 henrys
K 0.0089 microfarads	*0.0098 microfarads
$\frac{S}{K}$ 100	135

* The capacity as measured with direct current gave a similar result in this case.

The mean observed β was 0·0057. The maximum was 0·00627 and the minimum ·00522. The calculated β from the constants given in the first column is 0·0056. Although it will be seen that the mean figure obtained was very near to that usually taken, it should be noted that under different weather conditions the β varied nearly 10% above and below that figure, giving a total variation of nearly 20%. The cause of this variation was the different weather conditions which prevailed at various times, but it is also important to note that in these experiments the leakance at alternating current frequencies was considerably greater in very fine weather than that obtained by D.C. measurements. This is illustrated by Fig. 2. It will be seen that when the direct current loss was great, the difference between the D.C. readings and A.C. readings was not great, but when the D.C. insulation was very high, the A.C. insulation was never correspondingly high. This is shewn in another form in Fig. 3, where the ratio $\frac{S}{\frac{1}{\Omega}}$ is given for different weather conditions. The result is interesting, but more experiments are required before the ratio there shewn can be accepted as a general property of aerial lines. From the practical point of view this difference in A.C. and D.C. measurements loses much of its importance from the fact that when the D.C. insulation is well under one megohm, the difference between A.C. and D.C. measurements is relatively small. It is a point towards the lower limit of insulation that determines the practical value of a circuit

EXPERIMENTS ON 300 LB. AERIAL LINES.

VARIATION OF THE RATIO $S : \frac{1}{\Omega}$ WITH VARIATION OF $\frac{1}{\Omega}$

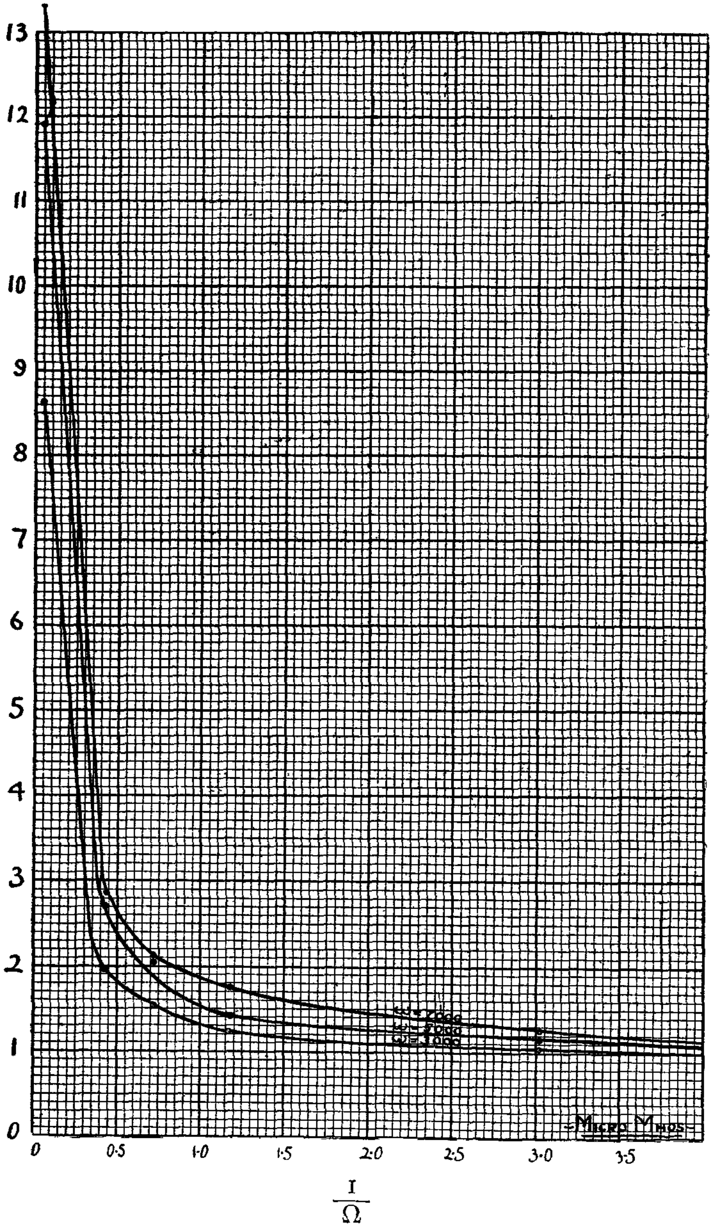


FIG. 3

EXPERIMENTS ON 300 LB AERIAL LINES

RELATIONSHIP BETWEEN LEAKANCE AS MEASURED BY FRANKE MACHINE AND THE ATTENUATION CONSTANT. FREQUENCIES RANGING FROM $\frac{3000}{2\pi}$ TO $\frac{7000}{2\pi}$

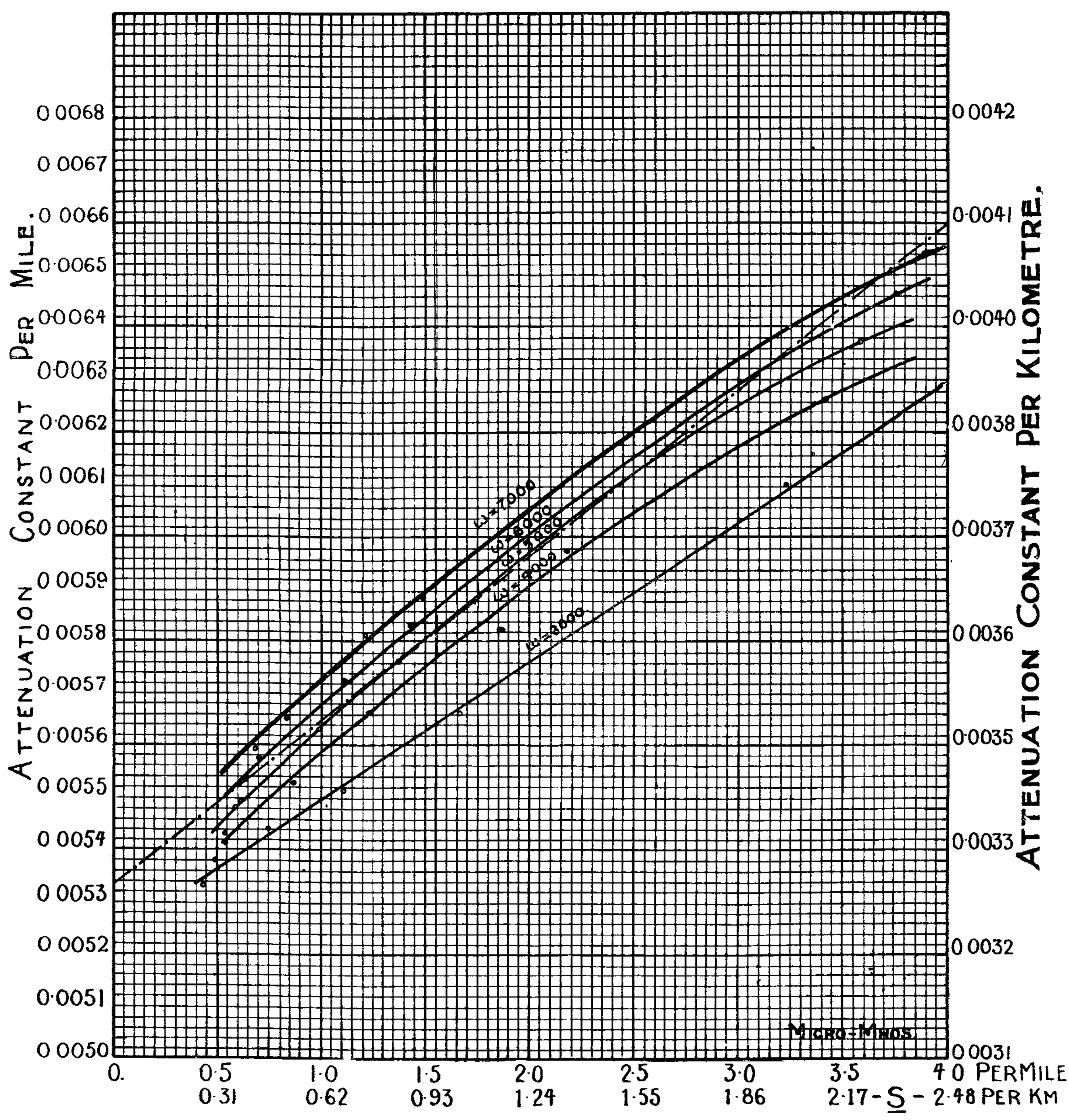


FIG. 4.

from the point of view of transmission efficiency. The practical outcome of the test is that whereas 1 megohm per mile is usually taken as the mean figure, it was found in these tests to be $\frac{2}{3}$ megohm, at a frequency such that $2\pi n = 5{,}000$. The importance of obtaining Insulators having a higher resistance in damp weather will be emphasised at a later stage. The observed and calculated variation of β with leakance at different frequencies is shewn in Fig. 4 and the observed figures agree over a large range with calculated figures.

A typical case of the variation of β with frequency on a given date and in the same weather conditions is shewn by Fig. 5, which gives the result of tests made during this series of experiments. The Resistance, Inductance, and Capacity shewed only very small variation with frequency, the variation in β was, however, greater, and this is largely due to the variation of leakance with frequency. This variation is shewn in Fig. 6. It should be noted, however, notwithstanding this variation, that speech over the unloaded circuits experimented upon was very clear in articulation as compared with underground circuits, where the distortion is much greater.

EXPERIMENTS ON AERIAL LOADED CIRCUITS.

GENERAL OBSERVATIONS.

The aerial circuits selected for loading were two 300-lbs. copper circuits between Leeds and London, the length of the aerial section, in the case of each circuit, being 200 miles. These circuits were selected as being free from underground sections, directly accessible from London, and of equal gauge. There are also on the same poles two other circuits exactly similar in every respect, so that it was possible to compare the loaded with the unloaded circuits in different weather conditions. It was not possible to obtain circuits of small gauge to fulfil all these conditions. A mean figure for the constants of the circuits when unloaded has already been given.

The attenuation of these circuits as measured by alternating currents at 800 periods per second is ·0057, and this agrees with calculated values very nearly.

EXPERIMENTS ON UNLOADED 300 LB. AERIAL LINES

VARIATION OF β WITH FREQUENCY

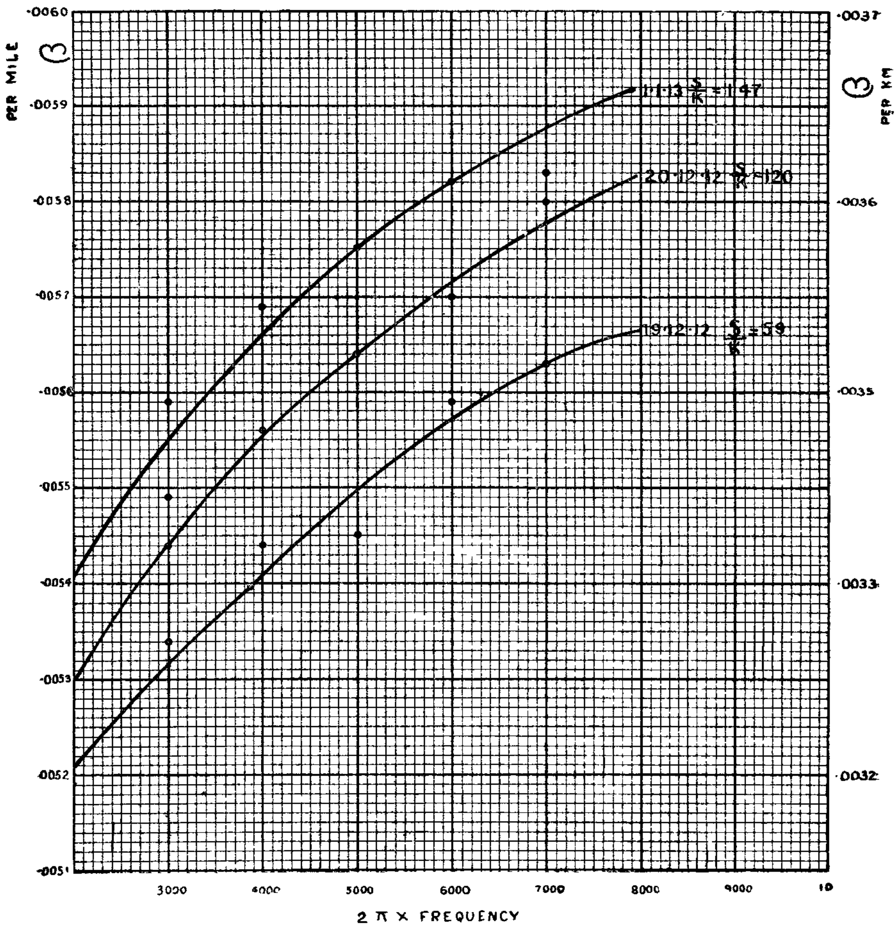
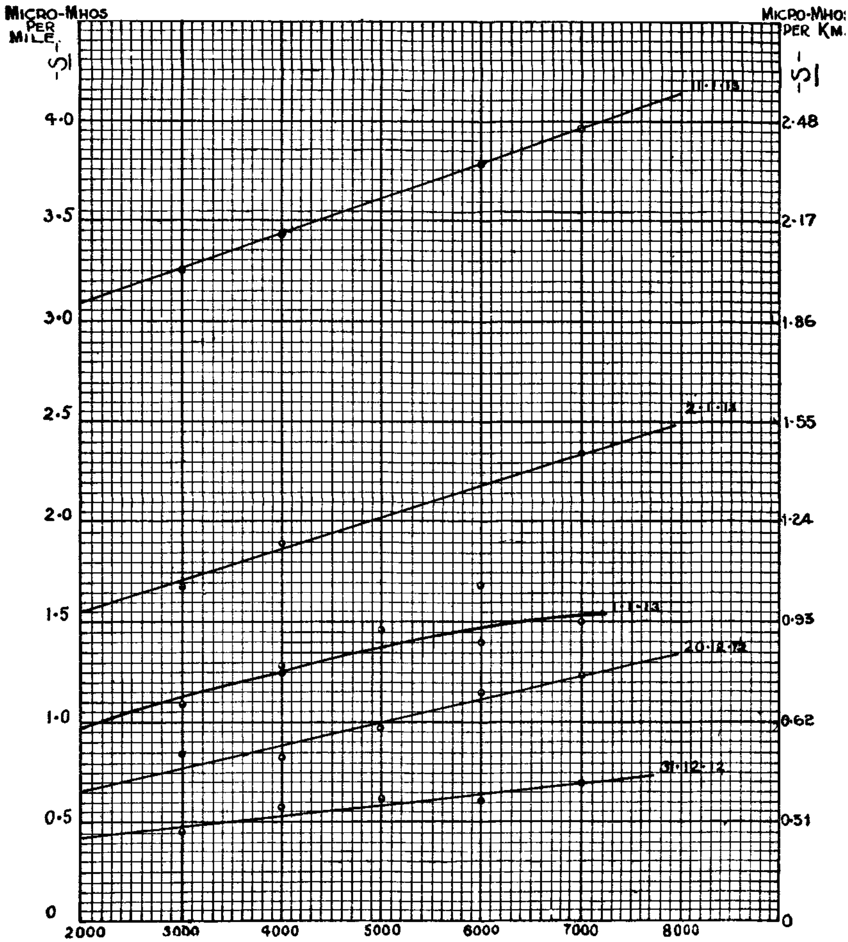


FIG. 5

— EXPERIMENTS ON 300 LB. AERIAL LINES —

VARIATION OF LEAKANCE WITH FREQUENCY ON VARIOUS DATES



2 π × FREQUENCY.

FIG. 6.

It was known that the Western Electric Company stock two types of loading coils specially manufactured for aerial loading. These coils have an inductance of 265 millihenrys, and 160 mill henrys respectively, and are hereafter called Coil No. 1 and Coil No. 2. The relation of effective resistance (R_1) to Inductance (L_1) in the coils is such that $\frac{R_1}{L_1} = 25$ at a frequency of 800 cycles per second, for both coils. The usual spacing for the former is 8 miles apart, and for the latter, twelve miles apart. In order to find whether these coils are suitable for loading the lines referred to, it will be well to inspect the formulæ already given. The attenuation constant is $\left(\text{from (12)}\right)$

$$\beta = \frac{R + R_1}{2}\sqrt{\frac{K}{L}} + \frac{S}{2}\sqrt{\frac{L}{K}} \qquad \text{(18)}$$

and in this form it is evident from the right hand portion of the formula that the leakance and inductance combined, may materially increase the attenuation of a circuit when L is large, or, in other words, in a loaded circuit, hence the importance of knowing the value of S.

From formula (11), the best Inductance is

$$L = \frac{6{\cdot}7}{25 + \frac{S}{K}}$$

where 6·7 is the circuit resistance and $\frac{R_1}{L_1} = 25$. If the mean $\frac{S}{K}$ found during the tests on unloaded circuits be taken, *i.e.*, $\frac{S}{K} = 135$, then the above formula gives 0·042 henrys per mile as the best inductance to be used for loading this circuit. Now the inductance per mile obtained by using Coil No. 1 is $\frac{0{\cdot}265}{8} = 033{\cdot}1$ henrys per mile. To this must be added the natural inductance of the circuit, *i.e.*, ·0038 henry, so that the total inductance used per mile was 0·037 henry in this case. It may be verified from formula (14) that the β obtained by the use of this inductance is within 1% of the best obtainable if the constants taken be accepted. During the experiments, however, an $\frac{S}{K}$ as high as 350 was observed and, in that case, the best loading is $L = \frac{R}{25 + 350} = 0{\cdot}0179$ henry. The inductance obtained by the use of Coil No. 2 was $\frac{\cdot 160}{12} + 0038 = 0171$ henry, and from formula (14) this gives practically the minium β for $\frac{S}{K} = 350$ and $\frac{R_1}{L_1} = 25$. It will be seen, therefore, that Coil No. 2 is suitable for use on lines with relatively low insulaton if the attenuation constant only be considered. It is, however. of great importance in the case of aerial circuits to consider also the characteristic impedance in the case of loading on account of its relative magnitude as compared with the impedance of other circuits.

The calculated characteristic impedance of the aerial circuits in question when loaded with Coil No. 1 is $1900\overline{\backslash 0^{\circ}24'}$
and of „ „ 2 „ $1300\overline{\backslash 1^{\circ}36'}$
whilst the same circuit, unloaded, gives $637\overline{\backslash 8^{\circ}55'}$

The significance of the characteristic impedance will be seen later when considering terminal losses, etc.

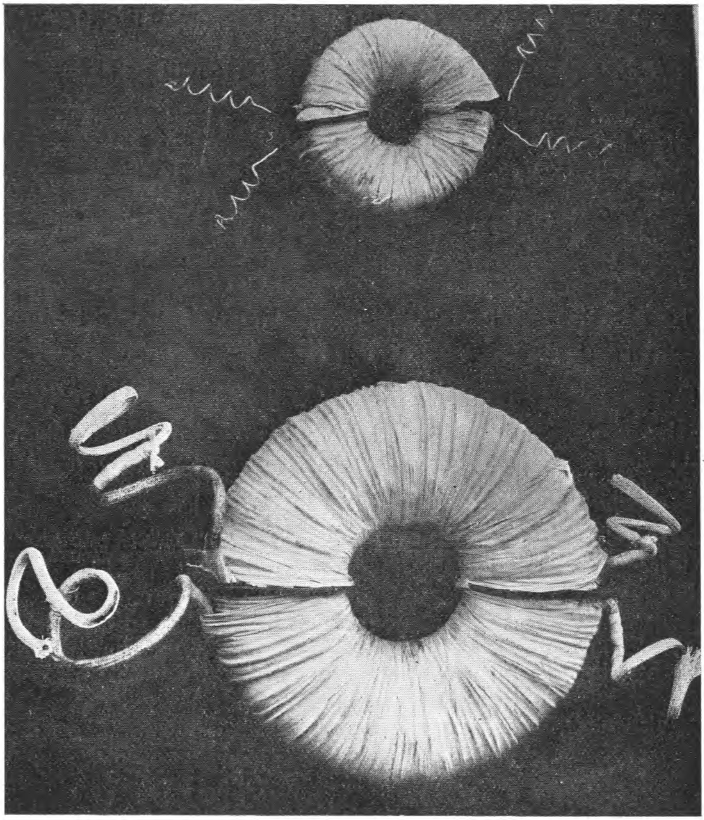
In view of the humidity of the atmosphere in this country. it was decided to load one of the circuits referred to with Coil No. 1 and a second circuit with Coil No. 2 and to determine by experiment which was the more suitable coil for the prevalent weather conditions. Accordingly, London-Leeds No. 6 Trunk Circuit was equipped with Coil No. 1, spaced 8 miles apart, and London-Leeds No. 7 Trunk Circuit with Coil No. 2, spaced 12 miles apart.

MECHANICAL DETAILS AND FITTING OF COILS AND LIGHTNING PROTECTORS.

The leading coils are toroidal in shape and are enclosed in strong iron cases specially adapted for attachment to an auxiliary arm which is fitted at the top of the pole.

Fig. 7 shews an aerial coil and an ordinary underground coil, both being unenclosed, the relative sizes of the coils being to scale. It will be observed that the aerial coil is much larger than the other, this is to permit of a lower effective resistance being obtained owing to the fact that aerial loaded circuits have generally a much less ohmic

RELATIVE SIZES OF LOADING COILS.



$\frac{R_1}{L_1} = 50$

$\frac{R_1}{L_1} = 25$

FIG. 7.

resistance per mile than underground circuits, and it is therefore considered necessary to keep the added resistance as low as possible. In this case the larger coil has half the effective resistance of the smaller one. Other considerations, however, require to be taken into account and these will be dealt with later. The completed coil, ready for attachment to the poles, is shewn in Figs. 8 and 9, and the

AERIAL LOADING COIL IN CASE.

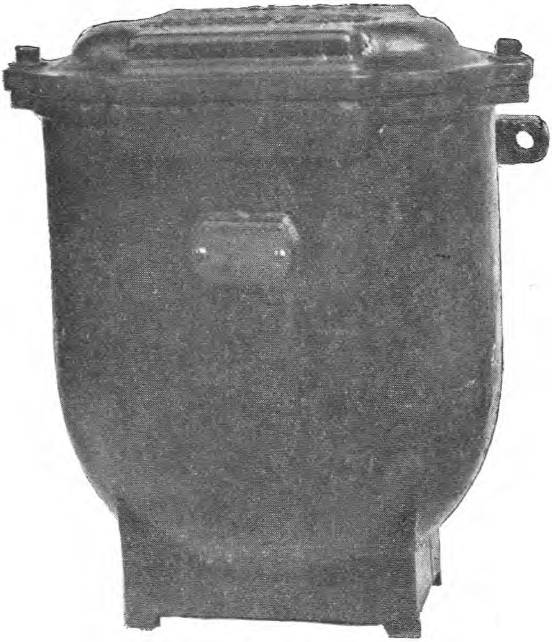


FIG. 8.

lightning protector is shewn in Figs. 13 and 14. It is necessary that every coil should be separately protected and that the protector should be of a reliable form and such as will stand weather exposure. The protector consists of discs of non-arcing metal and was supplied by the Western Electric Company. In practice one fault only has occurred in a Protector since the coils were installed about 18 months ago. A complete pole fitting, as arranged by the Engineer-in-Chief, is shewn in Fig. 10. Each loaded wire is terminated at every loading point. Each complete loading coil and case weighs approximately $\frac{3}{4}$ cwt. and is fixed on an

AERIAL LOADING COIL WITH LEADS.

FIG. 9.

LIGHTNING PROTECTOR FOR USE WITH LOADING COILS.

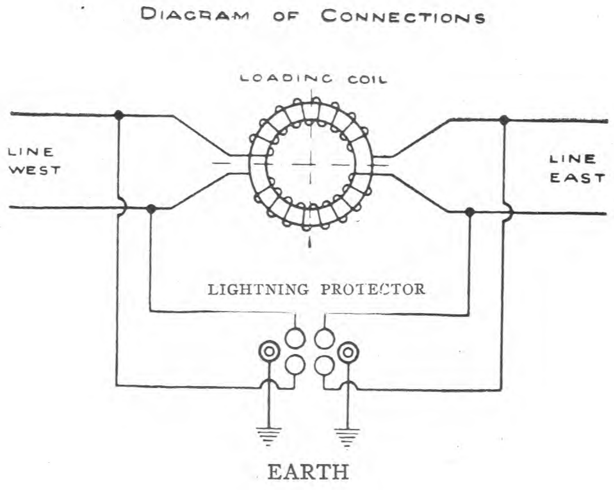


FIG. 13.

auxiliary G I support bolted to the back of the pole (Fig. 15). The leads are of vulcanised India rubber, and are cleated to the pole as shewn in the figure. (The V I R leads will be lead covered in future and the method of fitting somewhat altered. See Fig. 17).

EXPERIMENTAL OBSERVATIONS ON AERIAL CIRCUITS.

DETAILS OF TESTS.

The tests made were of two distinct kinds :—

(1) Measurements by an Alternating Current Machine ;

(2) Speech tests by comparison with a Standard Cable.

These two sets of results should agree with each other

LIGHTNING PROTECTOR.

T – 533 – B

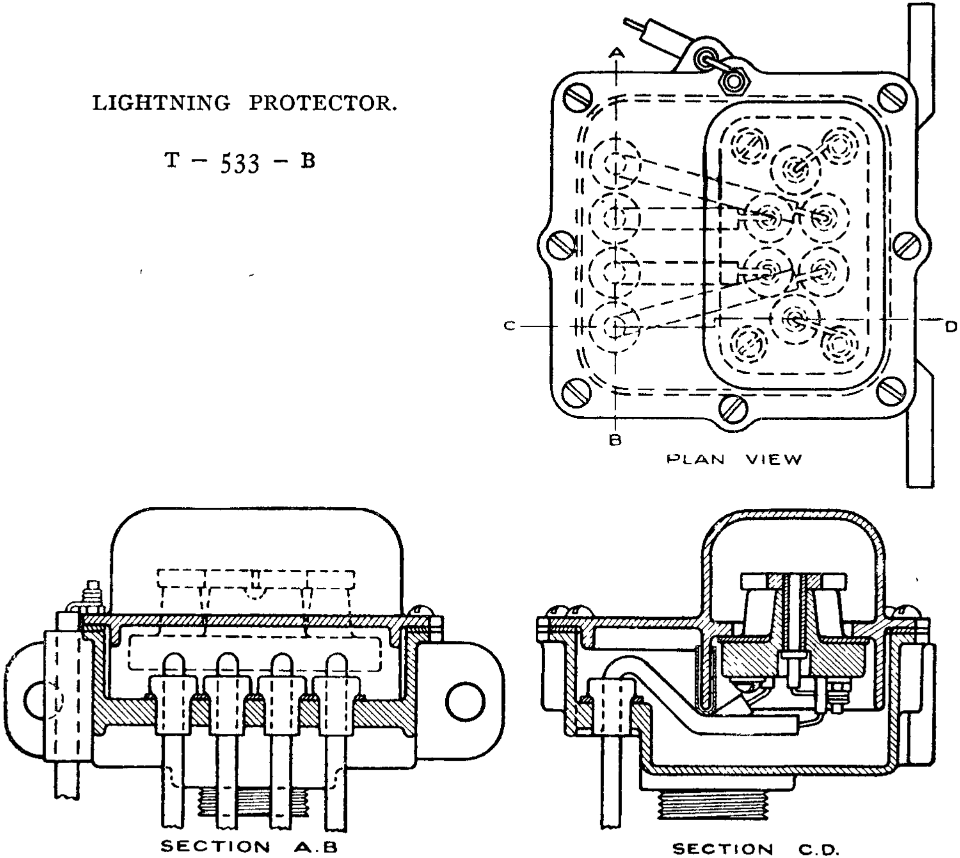


FIG. 14.

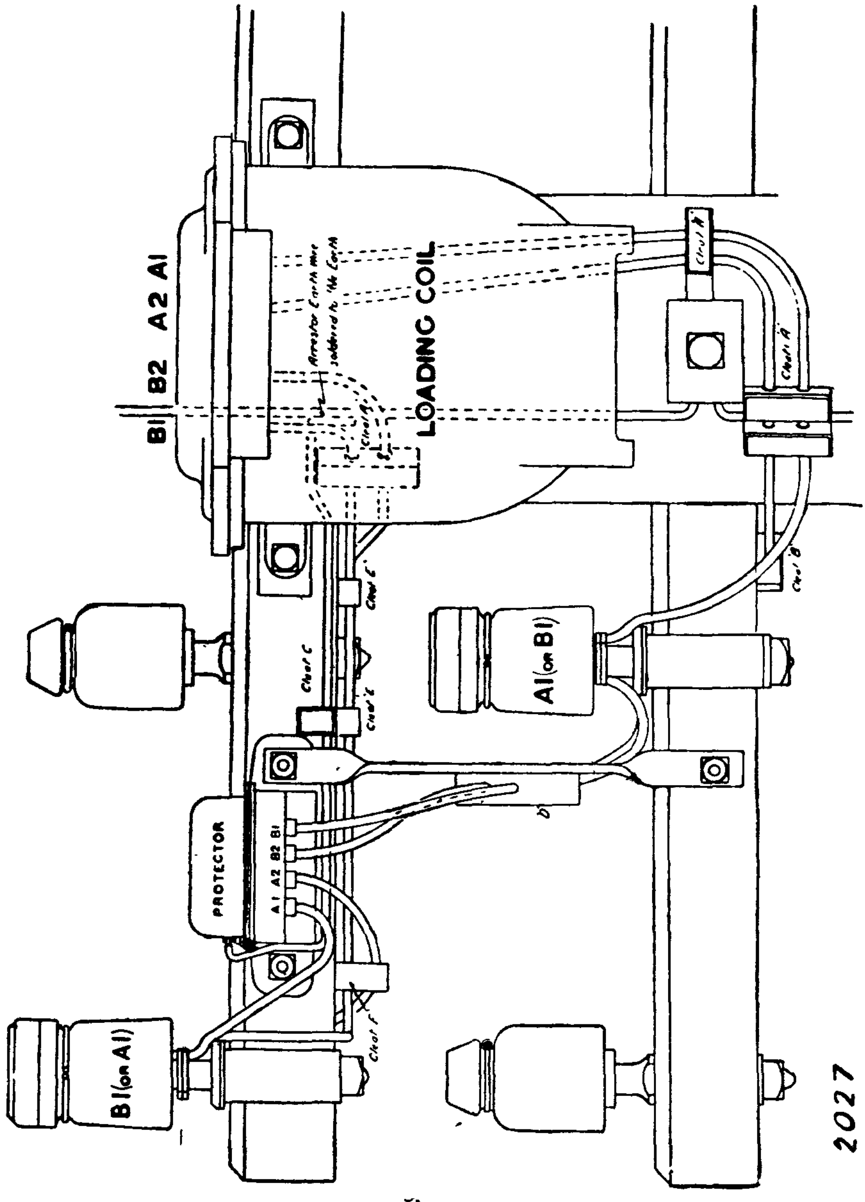

B1 B2 A2 A1
LOADING COIL
PROTECTOR
A1 A2 B2 B1
B1(or A1)
A1(or B1)
2027

FIG. 10.

AERIAL LOADING COIL FITTED TO POLE.
PROTECTOR MOUNTED ON CROSS ARM.

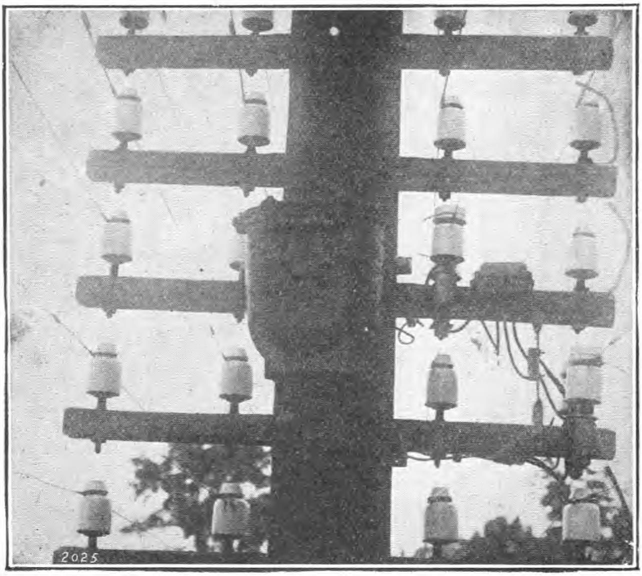


FIG. 15.

and with calculation, if the suppositions on which the tests are based are correct.

The relative behaviour of the loaded and unloaded crcuits was also kept under observation from the point of view of reliability in different weather conditions.

The circuits tested were London-Leeds Nos. 8 and 9 unloaded trunks, which are similar in all respects to London-Leeds Nos. 6 and 7 loaded trunks, and they were continually compared with them.

The principal point selected for the tests was St. Albans, due to the fact that it is readily accessible from London and very near the end of the open lines.

The manner in which the β of the unloaded circuits (Nos. 8 and 9 Trunks) and also the two loaded circuits (Nos. 6 and 7 Trunks) might be expected theoretically to vary with different values of S is shewn in Fig. 11. It will be seen

AERIAL LOADING COIL.

LATEST METHOD OF FIXING ON POLE USING V.I.R. LEAD-COVERED LEADS.

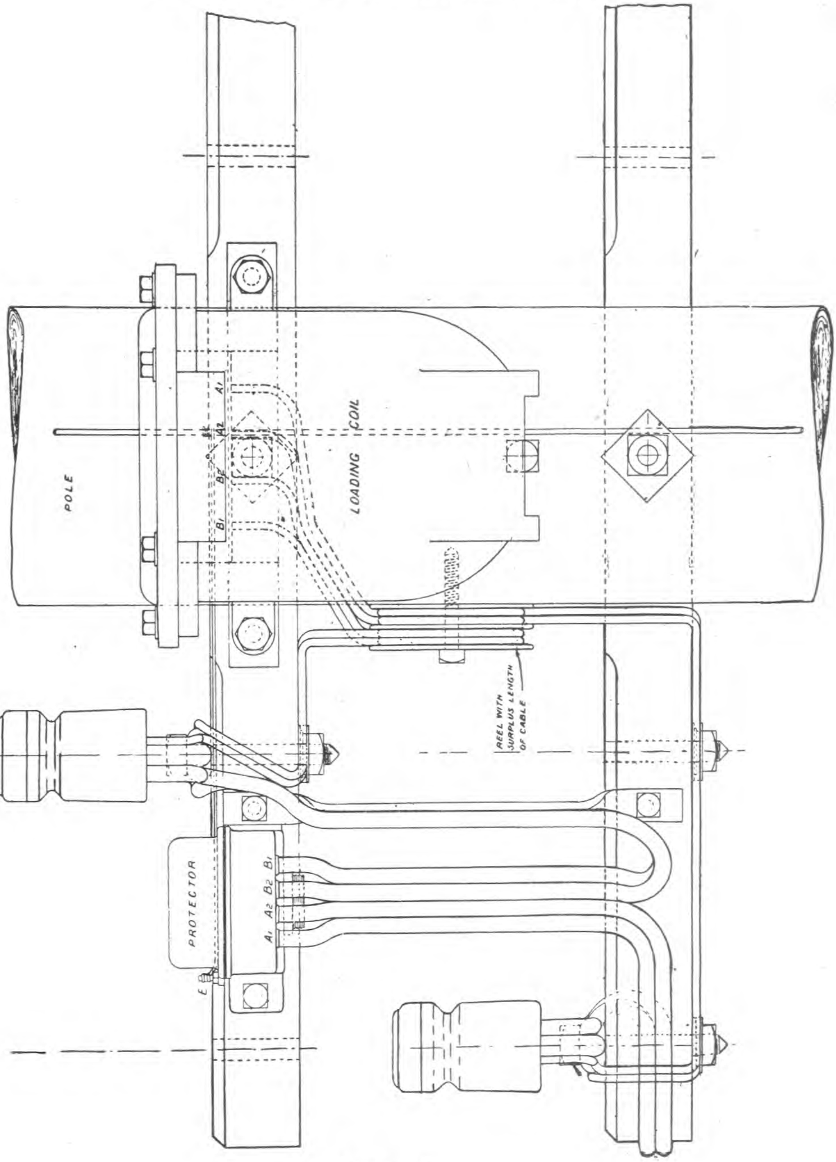


FIG. 17.

MEM.—There are two insulators on each double spindle. It is only possible to shew one of each in the drawing.

that β is plotted against $\frac{S}{K}$ (the latter ratio being more convenient for reference than leakance alone). Generally speaking, when the leakance is small, the curve shews the more heavily loaded circuit to be the best of the three. For values of insulation between 400,000ω and 128 000ω per mile, however, the more lightly loaded circuit might be expected to give the better result, and below that figure the β is more favourable in unloaded circuits. In practice, the greatest value of leakance observed during the whole of the tests was such that $\frac{S}{K}$ was about 350, so that, so far as those lines are concerned, the loaded lines might have been expected to give the best result and this was found to be the case.

The following schedule shews attenuation constants on one unloaded circuit and each of the loaded circuits, during the month of April, 1913 :—

TABLE 1.
LONDON-LEEDS Nos. 6, 7 & 9 CIRCUITS.
A.C. MEASUREMENTS.

Locality of Test From St. Albans to	Distance in miles.	Circuit Number.	β Per mile.	Insulation Resistance measured with D.C. (megohms per mile).	S/K from measurements at 800 periods per second.
Worksop	131	6	00312	19	45
,,	,,	,,	00267	4.16	65
,,	,,	,,	00283	2.27	95
,,	,,	,,	00291	1.04	135
Doncaster	148	,,	00310	5.34	55
Leeds	189	,,	00283	81.3	44
Worksop	131	7	00390	38	43
Doncaster	148	,,	00390	6.24	55
Leeds	189	,,	00375	11.3	—
Doncaster	148	,,	00374	2.27	90
Leeds	189	,,	00372	1.51	110
Kettering	53	9	00555	2.01	95
Worksop	131	,,	00520	15.10	48
Doncaster	148	,,	00548	4.60	60
Leeds	189	,,	00601	—	—
,,	,,	,,	00614	1.51	110

A number of earlier measurements taken during the winter months on the two loaded circuits had to be cancelled owing to some unfortunate fault which was very difficult to trace, and which rendered the results taken on the loaded circuits during that period unreliable.

The results on the unloaded circuits are within 6% of the calculated curve. Three of the six measured results on the more heavily loaded circuit are in fair agreement with the calculated curves, the other three are not so satisfactory. As regards No. 7 circuit, the results shew a general resemblance in the values of β regardless of the value of $\frac{S}{K}$. In view, however, of the large slope of the loaded curves with different values of $\frac{S}{K}$ as shewn by Fig. 11, it will be seen that the results are generally of the correct order. It should be noted, however, that the calculations assume a uniform distribution of leakance along the whole line, whereas in practice it was on several occasions verified by test that the distribution of leakance over the London-Leeds circuits was very irregular, and this in conjunction with occasional inductive disturbances, probably accounts in great part for the discrepancies between observation and calculation.

It is worthy of note that in the experiments previously referred to as having been made to determine the line constants, the results were regular and consistent with calculation, but in that case the length of line was only 30 miles, and the leakance was therefore usually much more evenly distributed than in the case of the relatively long line to Leeds.

The measurement of the characteristic impedance of the loaded lines shewed some discrepancies as compared with calculation. On the other hand the unloaded line measurement agreed well with calculation.

SPEAKING TESTS.

A number of speaking tests were made from three points on the line, viz., Harby, the middle point of the line, and St. Albans and Leeds, practically at the ends of the open line.

AERIAL 300 LBS. CIRCUIT (3·48 MM. DIAM.)
CURVES SHEWING VARIATION OF ATTENUATION CONSTANT WITH INDUCTANCE & LEAKANCE

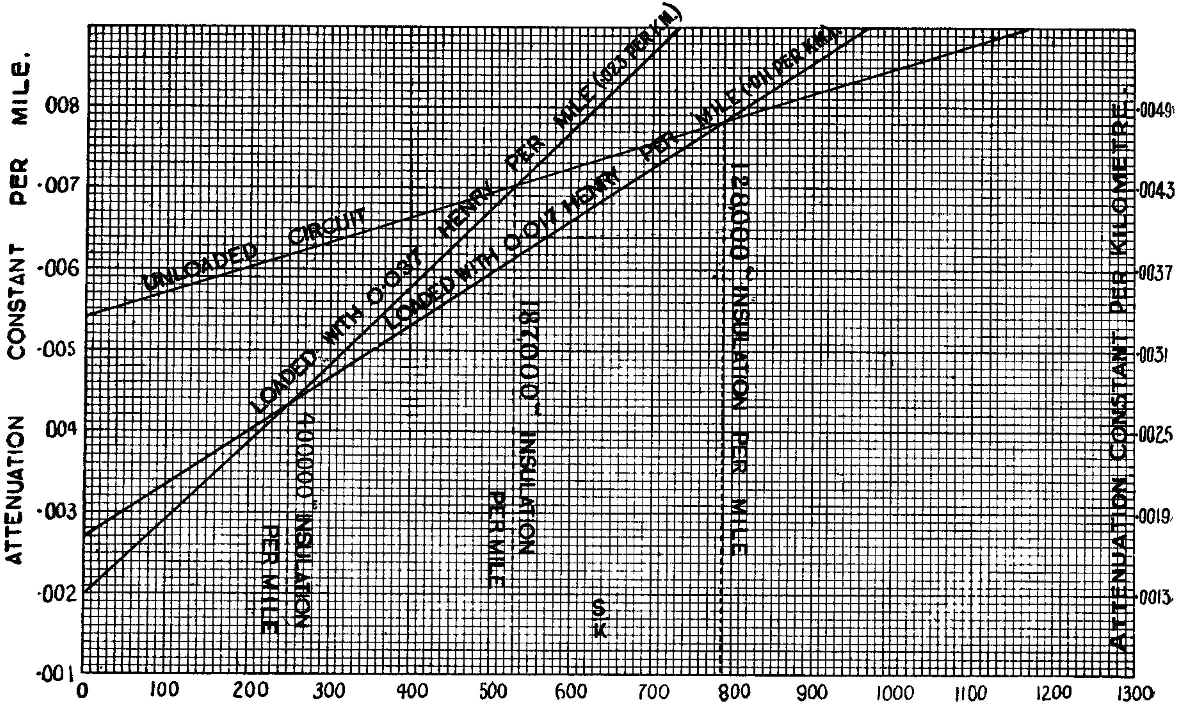


FIG. 11.

Before dealing with the observed results, it should be remarked that owing to occasional inductive disturbances added to the difference of "tone" noticed between speech on the aerial lines, and standard cable, and also the fact that in such tests personal opinion is a factor which must be recognised, a high degree of accuracy could scarcely be expected. If these points are borne in mind, the degree of concordance between calculation and experiment is fairly satisfactory.

Dealing first with the unloaded lines, the following observations with local battery telephones were noted on different dates:—

TABLE 2.

SPEAKING TESTS.

Circuits under test.	Length of circuit under test, miles.	Miles of standard cable observed	Calculated β taken from Fig. 11.	Experimental result in terms of β per mile.	Terminal.	
					Mileage at end of cable.	$\frac{S}{K}$
London	318	18	.0057	.0060	0	95
Leeds	184	$11\frac{1}{2}$	.0058	.0066	0	130
Nos. 8 & 9	378	22	.0058	.0062	0	120
Trunks	,,	20.8	.0058	.0058	10 m.s.c.	120
,,	,,	21	.0056	.0059	0	50

These results are representative of those generally obtained. Tests with common battery instruments are dealt with separately.

On the whole, the observed results are a little worse than the calculated ones. It should be noted, however, that slight disturbances on the line might be expected to increase the observed β and that the speaking apparatus at the end of the line was not brought into the calculation of the β to be expected. The observed result might, therefore, be expected to be a little higher than the calculated.

SPEAKING TESTS ON LOADED AERIAL CIRCUITS.

These tests were much more complex than those on unloaded lines owing to the high impedance of the loaded aerial circuits giving rise to marked terminal losses, and reflection between lines of very different impedance.

The tests made were of two kinds:—

(1) Tests of Nos. 6 and 7 (looped) from Harby and from St. Albans: to various points.

(2) Tests of 6 and 7 singly between St. Albans and Leeds,

both 6 and 7 in the latter test being compared directly with an exactly similar unloaded circuit, and the difference noted.

Fig. 12 shews the theoretical arrangement for test (2).

In the most conclusive test under this head (No. 2) a minimum length of 10 miles of standard cable was used as a terminal mileage at both ends of the circuit, and an additional standard artificial cable inserted in the line having the least standard cable value, as shewn in the diagram. The variable standard cable was altered until equality of speech was obtained in the two lines under test. The value of the unloaded circuit was afterwards obtained by testing a return loop between the same points.

The direct test of the circuits between Leeds and St Albans on two different occasions, and in the conditions indicated by the diagram, shewed a mean difference of $2\frac{1}{4}$ miles of standard cable between the heavily loaded circuit No. 6 and the unloaded circuit No. 9, and a difference of 1 mile of standard cable between the lightly loaded circuit No. 7 and an unloaded circuit No. 9. The mean value of the unloaded circuit with primary battery apparatus at the ends was $11\frac{1}{4}$ miles of standard cable, so that circuit No. 6 had the standard cable value $11\frac{1}{4} - 2\frac{1}{4} = 9$ and No. 7 the value $11\frac{1}{4} - 1 = 10\frac{1}{4}$ m.s.c.

Putting these results in tabular form, we have:—

TABLE 3.

SPEAKING TESTS.

Trunk Circuit under test.	Length, miles.	Terminal mileage.	Standard cable equivalent.
London-Leeds No. 6	189	10 m.s.c.	9
do. 7	,,	,,	$10\frac{1}{4}$
do. 9	,,	,,	$11\frac{1}{4}$

To ascertain whether these figures are in accordance with what should be expected they were compared with

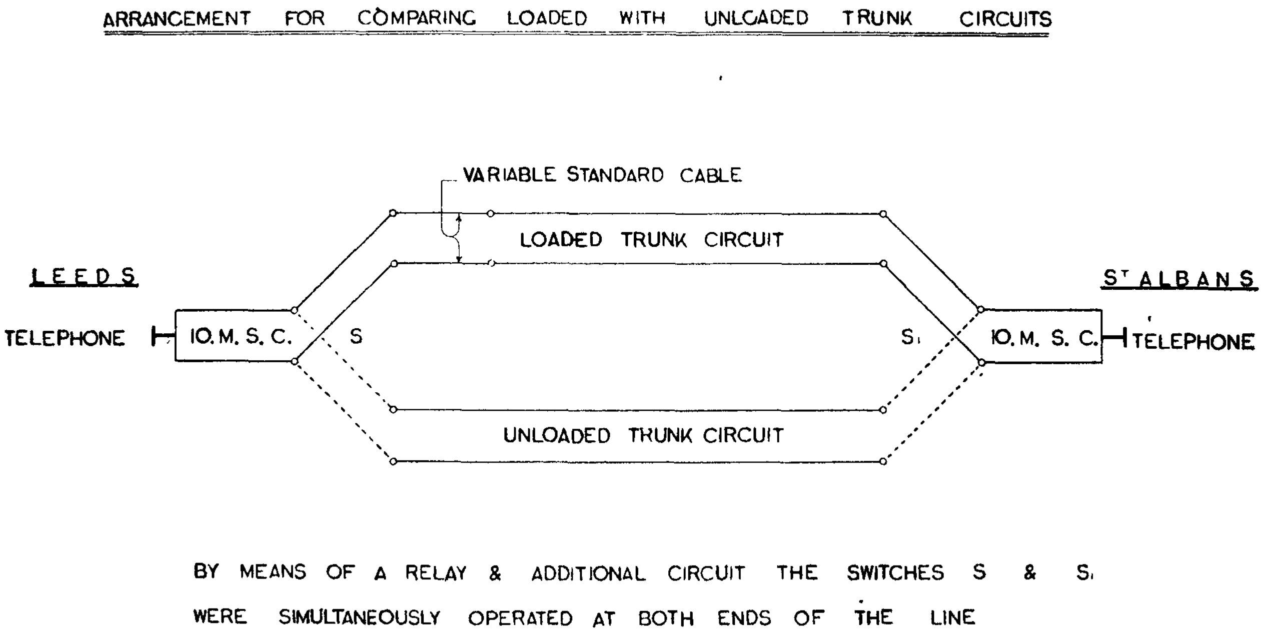

ARRANGEMENT FOR COMPARING LOADED WITH UNLOADED TRUNK CIRCUITS
VARIABLE STANDARD CABLE
LOADED TRUNK CIRCUIT
LEEDS
TELEPHONE
10. M. S. C.
S
S1
ST ALBANS
10. M. S. C.
TELEPHONE
UNLOADED TRUNK CIRCUIT
BY MEANS OF A RELAY & ADDITIONAL CIRCUIT THE SWITCHES S & S1 WERE SIMULTANEOUSLY OPERATED AT BOTH ENDS OF THE LINE

FIG. 12.

values measured by an alternating current machine at a frequency of 800 cycles per second.

The mean values of the measured attenuation constants are:—

LS 6 = 00291
„ 7 = 0038
„ 9 = 0057

The value of βl between St. Albans and Leeds as deduced from A.C. measurements was therefore:—

Trunk Circuit under test.		β		Length miles.		βl
Circuit No. 6	=	00291	×	189	=	0·55
„ „ 7	=	0038	×	189	=	0·718
„ „ 9	=	0057	×	189	=	1·08

Taking the β of the standard cable at 0·106, we have the following values of these tests expressed in standard miles:—

TABLE 4.

Trunk Circuits under test		Standard Cable Equivalent.
Circuit No. 6	=	5.2
„ „ 7	=	6.77
„ „ 9	=	10.22

In the case of LS 6 and 9 these are exactly the results which might be expected if $\frac{S}{K}$ were 100. LS 7, however, would only give 6·2 miles of standard cable at that figure. Now the weather was such during the standard cable tests under consideration that the value of $\frac{S}{K}$ is a reasonable one, so that, from that point of view, the comparison is valid. If nothing but the attenuation constants required to be taken into account these figures ought to agree with those in table 3. There are, however, other considerations which it will now be useful to discuss.

We have here the case of a short line (electrically speaking) in the middle of a relatively long one of a different kind. Theoretically, if we take the reflection factor

$$M = \frac{Z_0}{Z_r} + \frac{Z}{Z_0}$$

where Z_0 and Z_r are the characteristic impedance of the loaded cable and the standard cables respectively; then, in such cases we ought not to obtain the βl which would be obtained if the inserted line were long and uniform, but instead of that value we should get

$$e^{\beta l}[\tfrac{1}{2} + \tfrac{1}{4}M + e^{-2\beta l}(\tfrac{1}{2} - \tfrac{1}{4}M)]$$

Where β is the attenuation constant of the loaded line. The first two terms in the formula shew the loss at the first point of junction, and the remaining part shews the reflection from the second junction. It will be seen that different values are obtained from different lengths of circuit, very short lengths having only small loss (see Devaux Charbonnel—" Les Cables Telephoniques Sous-Marins " in " La Lumiere Electrique," 15th June, 1912. For full treatment see also Professor Breisig's " Theoretische Telegraphie," page 302).

The apparent loss in the case of circuit No. 6, if the measured β at 800 frequency be accepted, is 9 − 5·2 = 3·8 M.S.C. (*i.e.*, the difference between the values in tables 3 and 4).

In the case of circuit No. 7, it is 10·25 − 6·77 = 3·48 M.S.C., and in the case of circuit No. 9, it is 11·25 − 10·2 = 1 mile of standard cable. Similar losses were noted by three observers when testing loops from St. Albans to Leeds and back, instead of single lengths.

The tests on single lengths and loops agree fairly well with each other (allowing for some loss between circuits 6 and 7, when looped), but they do not shew good agreement with calculations made from the formula given above, and the single length of circuit. No. 6, for example, shews theoretically a loss of only 1·4 S.M. The precise reason for the discrepancy is not known; but the difficulties already mentioned should be borne in mind.

Tests were also made to ascertain the effect of extending the loaded lines by means of similar unloaded ones.

The mean of a number of tests gave, in round figures, an average loss of 2 miles of standard cable at each junction of the unloaded open lines with the loaded ones. The loss expected was about 1½ M.S.C.

The marked losses found in connection with aerial loading, point to the conclusion that terminal transformers are desirable. These transformers are so designed that the impedance of the two windings of the transformer are suitable for working in connection with the loaded and unloaded lines respectively, they can of course be varied to meet requirements.

By the use of such transformers the losses at points of junction, with heavy leading, can be reduced to approximately 1 standard mile.

The tests shewn in the following Table were made to determine the effect of the Standard Common Battery Set in connection with loaded lines. This combination is very frequent in practice. It will be seen that the loss on zero loop is considerable Figures for local battery sets are also given. In the case of these sets, calculation, assuming zero loop shewed fair agreement with observed results.

TABLE 5.

SPEAKING TESTS.

Comparison of losses on Common Battery and Local Battery sets in connection with loaded Aerial Lines.

Trunk Circuits under test.	Length in Miles.	Terminal Line.			Observed Miles of Standard Cable.			
						Common Battery Telephone.		
		Type of Line.	Length at each end.	Miles of Standard Cable.	Local Battery Telephone.	500 ohm loop.	300 ohm loop.	0 ohm (zero) loop.
London-Leeds 6 and 7, Loaded Circuits.	378	—	0	0	20	16.1	18.1	21.4
do.	,,	Standard Cable	10	20	16	—	16.3	—

EFFECT OF IMPEDANCE OF APPARATUS.

It is desirable to briefly draw attention to the fact (although this will not be new to many) that high impedance lines, such as the loaded lines in question, require special care in dealing with the apparatus used in connection with them. Experiments have fully demonstrated the fact that several relatively low impedance telephones, bridged across the loaded lines in the manner of an omnibus circuit ($\frac{1}{25}$ Induction Coils and 60ω Receivers) gave rise to much greater losses than the same number of sets of apparatus of higher impedance ($\frac{1}{150}$ Induction Coils and 150ω Receivers).

It is specially important that the bridged apparatus in operators cord circuits should be of suitably high impedance. Experiments made on some of the ordinary trunk cords used throughout the Country, however, shewed them to be well adapted for the work.

DEDUCTIONS AND CONCLUSIONS.

It is evident from calculation, from alternating current measurements, and speaking tests, that the loading of open lines of sufficient length leads to improvement of speech efficiency if the degree of insulation found in these tests can be maintained. Further, as no trouble has been experienced up to the present and the line used for the test is a normal main line, the prospect is distinctly favourable. An extension of the experiment is, however, being arranged to obtain more extensive data in different climatic conditions.

AGREEMENT BETWEEN CALCULATION AND EXPERIMENT.

The calculated figures for β on both loaded and unloaded lines, when compared with measurements at alternating current frequencies, shew a fair degree of agreement when the circuit constants are accurately known.

Speaking tests on unloaded Aerial circuits shewed slightly higher results than indicated by calculation of β alone, the result varying somewhat with the apparatus used. With the most suitable apparatus the loss was very small.

The losses found in loaded aerial circuits are so large and difficult to predict, that working figures should be obtained from a combination of alternator and voice tests. The losses found in various conditions in these tests are indicated in the paper. They are given as only approximately correct, but as giving a useful indication of the order of losses to be expected.

QUALITY OF SPEECH ON LOADED AERIAL CIRCUITS.

Direct comparison between both the loaded and the unloaded circuits (of exactly the same type) shewed a distinct degradation of articulation on the loaded circuits. The quality of speech on the latter was, however, commercially satisfactory.

The observed result was what might have been expected in view of the known variation of the resistance of loading coils, and of leakance, with frequency, and the effects of lump loading.

IMPROVEMENT OBTAINED BY LOADING AERIAL LINES.

The efficiency obtained by aerial loading of circuits of the type dealt with in this paper in the best weather approximates to reducing the value of β by one half (excluding end effects). The mean improvement in this climate would probably be a reduction of the unloaded value of β to the extent of 40% and in cases of low insulation 30%—excluding end effects, but taking into account the fact that the unloaded circuit would also be subject to variation.

In order to obtain any marked improvement in this Country in wet weather (say $\frac{S}{K} = 250$) where only 30% improvement is obtainable, a considerable length of line would be necessary. This might be reduced to a minimum by small gauge wires. It appears clear that aerial loading has a much greater scope in larger countries than in England.

RELATIVE ADVANTAGES OF OVERHEAD AND UNDERGROUND LOADING.

The minimum β in both cases may be found with sufficient accuracy by the formula

$$\beta m = \sqrt{KR\left(\frac{R_1}{L_1} + \frac{S}{K}\right)}$$

The capacity K in an overhead line may be taken as 0·01 mf. and in an underground A S P cable at 0·065 mf. per mile (the small capacity of the coil is negligible).

For underground work a coil having an $\frac{R_1}{L_1} = 35$ is available, and it is reasonable to suppose that a value of $\frac{S}{K} = 20$ can be obtained by expert manufacturers.

Comparing the formula in the two cases, we have

$$\text{U/G} \quad \beta m = \sqrt{R \times 0{\cdot}065 \times 10^{-6} \times (35 + 20)}$$

$$\text{Open } \beta m_1 = \sqrt{R \times 0{\cdot}01 \times 10^{-6} \times \left(25 + \frac{S}{K}\right)}$$

$$\text{that is } \frac{\beta m}{\beta m_1} = \sqrt{\frac{R \times 6{\cdot}5 \times 55}{R \times 1 \times \left(25 + \frac{S}{K}\right)}}$$

$$\text{that is } \frac{\beta m}{\beta m_1} = \sqrt{\frac{357{\cdot}5}{\left(25 + \frac{S}{K}\right)}}$$

that is if the value of $\frac{S}{K}$ in the aerial line becomes 332·5 the underground circuit is theoretically as efficient as the overhead one. It has been shewn that this actually happens so that underground circuits may be made as efficient as overhead *loaded* lines, in the minimum effic ency conditions obtaining during Winter in this country. The underground circuit has the advantage that it would have a lower characteristic impedance, and would be much more reliable. It would, however, in many cases probably be much more costly. In practice the small gauge circuits in underground cables could not be economically loaded to give the minimum β, but in the case of heavy gauge circuits this is quite practicable. There is also a much greater probability of successful superimposing on underground circuits than overhead lines.

The prospect of long distance underground circuits is attractive and is certain to be closely studied in the near future.

The above comparison is made on the supposition that overhead lines are very materially inferior to underground lines from the point of view of insulation. In view, however, of the fact that overhead lines would be distinctly superior to underground cables if their insulation could be increased, attempts will certainly be made to effect an improvement.

RELATIVE ADVANTAGES OF HIGHLY EFFICIENT AND LESS EFFICIENT LOADING COILS IN LEAKY CIRCUITS.

If it be assumed that a coil must be designed for a circuit having a definite value of leakance say, $\frac{S}{K} = 250$, it is desired to investigate what would be the effect of using a coil having a time constant of say 25 (the present aerial line coil) and one having a time constant say 35 (available for underground purposes).

This may be seen by the use of the formula

$$\beta m = \sqrt{KR\left(\frac{R_1}{L_1} + \frac{S}{K}\right)}$$

Case (1) Let $\beta m = \sqrt{KR(35 + 250)}$

Case (2) „ $\beta m_1 = \sqrt{KR(25 + 250)}$

$$\text{Then } \frac{\beta m}{\beta m_1} = \sqrt{\frac{275}{285}} = 1{\cdot}018$$

That is a little less than 2% increase in the minimum β obtainable is involved by the use of the coil having the higher time constant. The latter coil is, however, of appreciably smaller dimensions than the one used in these experiments, and there is no doubt that its use would lead to material economy in such a case.

THE MOST SUITABLE INDUCTANCE FOR LOADING ENGLISH AERIAL LINES.

This depends on the amount of leakance and the gauge of the wires to be loaded. The average effective $\frac{S}{K}$ was found in these tests to be 135 and the minimum 350. It is important to provide for the most beneficial effect of loading at the time the circuit most requires improvement, *i.e.*, when the leakance is greatest, but this precaution should not be pushed to abnormal limits, seeing that the value of $\frac{S}{K}$ only fell to 350 on one occasion. A mean between 135 and 350 would probably be a good compromise, in that case $\frac{S}{K} = 242{\cdot}5$ or say, in round figures, 250. Even then it will be seen from Fig. 11 that the 265 m.h. coil still gives as good a result as the lighter loading coil (with $\frac{S}{K} = 250$) and as it is superior for all less values of $\frac{S}{K}$ it is to be preferred. The ideal loading, however, for $\frac{S}{K} = 250$ with a coil of value $\frac{R_1}{L_1} = 25$ is 25 millihenrys per mile in the case of the particular circuit under investigation. It is not economical, as a rule, to design special coils for each circuit. If the whole range of aerial conductors be considered in connection with the $\frac{S}{K}$ values found in these experiments, it will be seen that one or other of the coils used in these tests fairly meets the requirement from the point of view of Inductance.

The investigation dealt with in this paper was carried out in the Engineer-in-Chief's Research Section, G.P.O. The study of the scheme and supervision of the tests was entrusted to me on behalf of the Engineer-in-Chief by Mr. Kempe, the then head of the Section, who was greatly interested in the experiments, and this also applies to Mr. Pollock, who has given every facility in connection with the matter.

A large part of the experimental work was carried out by Messrs. Robinson & Morice, and these gentlemen have given material assistance in connection with the more laborious calculations. Mr. Chamney has also taken a very active part in the tests. I have pleasure in tendering my cordial thanks to these gentlemen, the more especially as

nearly all the experimental work was necessarily performed after "office hours."

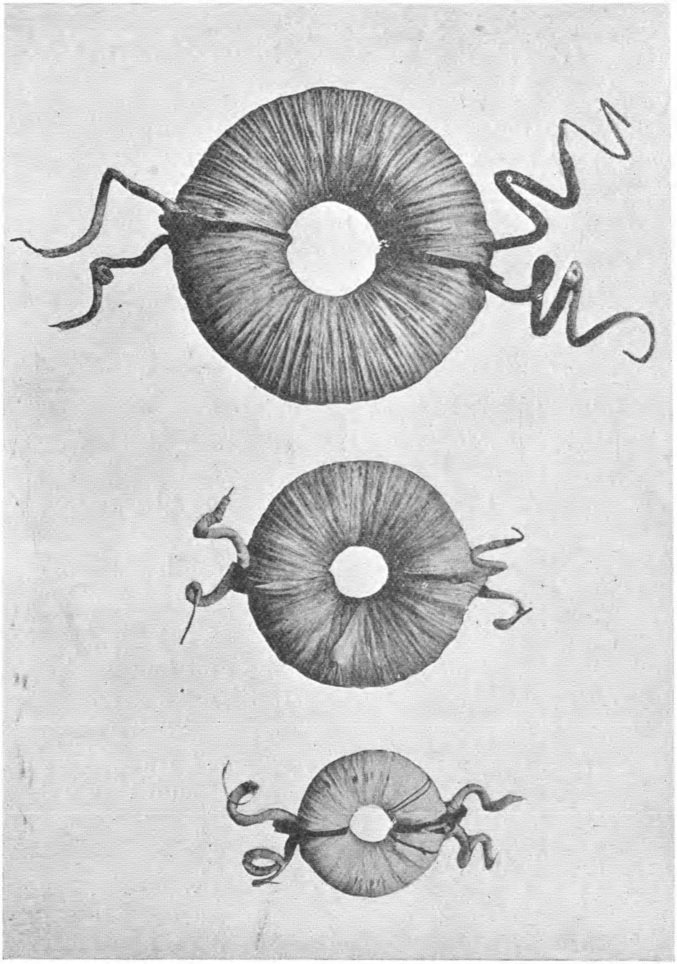
The Western Electric Co. have been good enough to provide photographs and drawings of the coils and protectors furnished by them. I am glad to have the opportunity of acknowledging these, and also the readiness with which any information in connection with the coils was furnished by the firm's representative, Mr. P. E. Erikson.

In conclusion, I wish to emphasise the growing importance of loading. The Department has at present not less than 13,000 loading coils actually installed in underground circuits and nearly 15 000 more are at present in process of installation in lead covered cables. These coils are generally spaced $2\frac{1}{2}$ miles apart, so that the Post Office will shortly own approximately 70,000 miles of loaded underground circuits, in addition to several submarine loaded cables and the aerial lines referred to in this paper. The advantages of loading are so considerable that it is certain that the principle will be widely applied in the future.

It is for this reason that I have attempted to simplify the proofs of the more general and widely used formulæ and have analysed some experimental results. I am convinced that some attempt is urgently needed to make the subject accessible to a wider range of readers, and I hope that this paper will be a step in that direction.

[Since this paper was read the Western Electric Co. have designed and manufactured for the use of the Post Office a number of aerial loading coils having an effective resistance of 35 ohms per henry $\left(\frac{R_1}{L_1} = 35\right)$ *at 800 periods per second (see page 46). The weight of these coils in their iron cases is two thirds of that of the aerial coils (made by the same firm) having an effective resistance such that* $\frac{R_1}{L_1} = 25$ *at 800 periods per second, as described in this paper. Fig. 16, taken from a photograph furnished by the Western Electric Co., shews how the new type of coil compares in size with those indicated in Fig. 7. The new coil is shewn in the centre in Fig 16.]*

COMPARATIVE SIZES OF LOADING COILS.



$\frac{R_1}{L_1} = 25$

$\frac{R_1}{L_1} = 35$

$\frac{R_1}{L_1} = 50$

Frequency
800 periods
per second.

FIG. 16.

APPENDIX I.

ELEMENTARY PROOF OF THE FOLLOWING TRANSMISSION FORMULÆ.

$$a = \sqrt{(R + jpL)(S + jpK)}$$ (The Complex Attenuation Constant).

$$Z_0 = \sqrt{\frac{R + jpL}{S + jpK}}$$ (The Characteristic Impedance).

The proof applies to a uniform telephone circuit which is long in the electrical sense, *i.e.*, a so-called infinite line.

Where R is the resistance of unit length of circuit:

S " " leakance " " " "

L " " inductance " " " "

K " " capacity " " " "

p = $2\pi n$ where n is the number of periods per second.

j = $\sqrt{-1}$ and indicates a rotation of 90°.

$a = j\alpha + \beta =$ The Complex attenuation constant, where α is the wave length constant and β is the real part of the Complex attenuation constant.

If dl is a very small fraction of the unit length, the following will be the electrical values for that small length of the circuit:—

$R \times dl$ = Resistance of a very small fraction of unit length:

$S \times dl$ = Leakance " " " "

$L \times dl$ = Inductance " " " "

$K \times dl$ = Capacity " " " "

If V_0 is the voltage at the beginning of the length dl and V_1 the voltage at the end of this small length, then the voltage drop in the length dl is $V_0 - V_1$, and similarly, in the next similar small length it may be written $V_1 - V_2$ where V_2 is the voltage at the end of the second small length dl.

Let $V_0 - V_1$ be called dv

$V_1 - V_2$ „ „ dv_1, etc.

Although the voltage in such a line falls continuously along its whole length, we may *suppose* it to be unchanged in the length dl if that length be small enough. Call the current in this small length (*i.e.*, at the beginning of the line) I_0. The impedance of unit length is $R + jpL$, and the impedance of the small length dl is $(R + jpL)\,dl$.

Now Volts = Impedance × Current

$$\therefore V_0 - V_1 = dv = (R + jpL)\, I_0\, dl \quad \text{.................}(1)$$

= Volts consumed in length dl.

But current is also lost in the same length, firstly, owing to direct leakage, and secondly, because the dielectric gives rise to a capacity charge. The leakage is $\dfrac{\text{Volts}}{\left\{\begin{matrix}\text{Insulation}\\ \text{Resistance}\end{matrix}\right.}$ = volts × leakance = $VSdl$. The capacity charge is $pK\, V_0 dl$. The amount of loss due to these causes is $I_0 - I_1 = di$ where I_0 and I_1 are the current at the beginning and end of the small length dl. Then

$$di = (V_0 S dl + jpK\, V_0 dl) = (S + jpK)\, V_0 dl \quad \text{......}(2)$$

Divide (1) by (2)

$$\text{We have } \frac{dv}{di} = \frac{(R + jpL) I_0 dl}{(S + jpK) V_0 dl}$$

$$\text{whence } \frac{dv}{di} \times \frac{V_0}{I_0} = \frac{R + jpL}{S + jpK} \quad \text{........................}(3)$$

But $dv = V_0 - V_1$ and if Z_0 is the characteristic impedance of the line, at any point, we have $V_0 = Z_0 I_0$ and $V_1 = Z_0 I_1$

So that $I_0 Z_0 - I_1 Z_0 = (I_0 - I_1) Z_0 = V_0 - V_1 = dv$

Again, seeing that $I_0 - I_1 = di \therefore (I_0 - I_1)\, Z_0 = di\, Z_0 = dv$

Therefore (3) may be written

$$\frac{diZ_0}{di} \times \frac{I_0 Z_0}{I_0} = \frac{R + jpL}{S + jpK}$$

$$\text{that is, } Z_0^2 = \frac{R + jpL}{S + jpK}$$

$$\text{and } Z_0 = \sqrt{\frac{R + jpL}{S + jpK}} \quad \text{.................... (4)}$$

Z_0 is called the charactersitic impedance of the circuit. Any electrically very long telephone circuit has this impedance.

Again, from (1) and (2)

$$dv\, di = (R + jpL)\, I_0 dl\, (S + jpK)\, V_0 dl.$$

$$\text{and therefore } \frac{dv\, di}{V_0 I_0} = dl^2 (R + jpL)(S + jpK)$$

$$\text{that is } \frac{di^2 \times Z_0}{I_0^2 Z_0} = dl^2 (R + jpL)(S + jpK)$$

$$\text{that is } \frac{di}{I_0} = dl\sqrt{(R + jpL)(S + jpK)} \quad \text{..... (5)}$$

That is to say, the ratio of the lost current in any short length dl to the current I_0 at the beginning of that section is a constant fraction. This may also be expressed as

$$\frac{I_0 - I_1}{I_0} = \frac{di}{I_0}$$

The current at the beginning of the second short section is I_1 and the current lost in that section is $I_1 - I_2$ if I_2 is the current at the end of the second short section.

$$\text{Now } \frac{I_1 - I_2}{I_1} = \frac{I_0 - I_1}{I_0}$$

for each $= dl\sqrt{(R + jpL)(S + jpK)}$ from (5) et seq.

In a third short section we should have

$\dfrac{I_2 - I_3}{I_2} = \dfrac{I_0 - I_1}{I_0}$ and so on to n short sections, and

since each of these ratios $= dl\sqrt{(R + jpL)(S + jpK)}$

therefore $n \dfrac{I_0 - I_1}{I_0} = n \times dl\sqrt{(R + jpL)(S + jpK)}$,

n being a number such that $n \times dl$ = unit length.

If we call the unit length 1, then $n \times dl = 1$, and we get

$$n\left(\frac{I_0 - I_1}{I_0}\right) = n \times \frac{di}{I_0} = \sqrt{(R + jpL)(S + jpK)}$$

That is to say, in any unit length, whatever it may be (an inch, a mile, or a kilometre), the ratio of the current lost in that length to the current at the beginning of it, is n times the fractional loss found for a very small length dl (where $n \times dl$ = unit length), and is measured by the quantity $\sqrt{(R + jpL)(S + jpK)}$, where R L S and K are the constants for unit length. This quantity is known as the complex attenuation constant. It is complex because it involves phase. The constant is usually denoted by a, so that our expression becomes

$$n \times \frac{di}{I_0} = a = \sqrt{(R + jpL)(S + jpK)}$$

From inspection of the formula it may be seen that it consists of a real and an unreal part, and it may be written

$$a = j\alpha + \beta = \sqrt{(R + jpL) S + jpK)} \quad \text{......(6)}$$

The two parts may be separated algebraically, and we get

$$\beta = \sqrt{\tfrac{1}{2}\sqrt{(R^2 + p^2L^2)(S^2 + p^2K^2)} + \tfrac{1}{2}(RS - p^2KL)} \quad \text{...(7)}$$

This is the attenuation constant, and

$$\alpha = \sqrt{\tfrac{1}{2}\sqrt{(R^2 + p^2L^2)(S^2 + p^2K^2)} - \tfrac{1}{2}(RS - p^2KL)} \quad \text{...(8)}$$

This is the wave length constant.

The electrical constants involved in α and β are the effective values at some definite frequency.

Those who desire the full working out of α and β will find this in Mr. Martin's Paper on "Loaded Underground Circuits," page 15.

It has been shewn that the ratio of the lost current in any short length, to the current at the beginning of that length is a constant fraction anywhere in a uniform and infinite line, although the current is constantly altering in magnitude throughout the line. A little consideration will shew that this is a case of the compound interest law, and that, consequently, the value of the current at any point is of the form $y = e^{-al}$ where y is the current and e is the base of the natural logarithms; also, a is the complex attenuation constant per unit lengh, and l is the number of units of length.

This may be proved as follows:—

Let the current at the beginning of the line be 1 and let a be the fraction of current lost in the first mile. At the end of this mile the current will be $1 - a$, and this is the current at the beginning of the second mile. The same fraction of this beginning current, *i.e.*, of $1 - a$, will be lost in the second mile as was lost in the first mile. So that at the end of the second mile the value of the remaining current will be $(1 - a)$ of $(1 - a) = (1 - a)^2$. Similarly, at the end of l miles it will be $y = (1 - a)^l$. If instead of taking so large a length as a mile for the unit, an exceedingly small fraction of a mile be taken; such that instead of losing the fraction a of the current per unit length, only the fraction $\frac{a}{n}$ (where n is a very large number) is lost, then at the end of the first mile the current will have the value $\left(1 - \frac{a}{n}\right)^n$ and at the end of l miles, it will be $\left(1 - \frac{a}{n}\right)^{nl}$.

It is known and can be verified, that if the algebraical quantity $\left(1 + \frac{1}{x}\right)^x$ be expanded by the Binomial theorem, and x then assumed to be a very large number, the Summation amounts to $2{\cdot}71828 = e =$ the base of the Natural logarithms.

To make use of this fact, we convert $\left(1 - \frac{a}{n}\right)^{nl}$ to the form $\left(1 + \frac{1}{x}\right)^x$. To effect this let $\frac{1}{x} = -\frac{a}{n}$ then

$n = -ax$. Substitute this value of n in $\left(1 - \frac{a}{n}\right)^{nl}$ and we get $\left(1 - \frac{a}{n}\right)^{nl} = \left(1 - \frac{a}{-ax}\right)^{-axl} = \left(1 + \frac{1}{x}\right)^{x \times -al}$ $= e^{-al} = y$.

If the current instead of being 1 at the commencement of the line has any value A, then e^{-al} becomes Ae^{-al}. But, as already shewn, $a = \beta + j\alpha$ and the expression becomes $y = Ae^{-(\beta+j\alpha)l}$. By means of this expression, knowing the value A of the current at the beginning of the line, its value y in magnitude and phase can be found at any distance l along the line. β and α are constants in the conditions supposed in this proof, l being, of course, variable.

APPENDIX No. 2.

In any circuit if $S = 0$ and pK is great in comparison with R, the circuit is distortionless.

$$\beta = \sqrt{\frac{1}{2}\sqrt{(R^2 + p^2L^2)(S^2 + p^2K^2)} + \frac{1}{2}(RS - p^2KL)}$$

in any circuit whatever.

If $S = 0$ this becomes

$$\beta = \sqrt{\frac{1}{2}\sqrt{(R^2 + p^2L^2)\, p^2K^2} - \frac{1}{2}p^2KL}$$

$$= \sqrt{\frac{1}{2}pKR\left(\sqrt{(R^2 + p^2L^2)\frac{p^2K^2}{p^2K^2R^2}} - \frac{p^2KL}{pKR}\right)}$$

$$= \sqrt{\frac{1}{2}pKR\left(\sqrt{\frac{(R^2 + p^2L^2)}{R^2}} - \frac{pL}{R}\right)}$$

$$= \sqrt{\frac{1}{2}pK\left[\left(R^2 + p^2L^2\right)^{\frac{1}{2}} - pL\right]}$$

$$= \sqrt{\frac{1}{2}pK\left[pL\left(1 + \frac{R^2}{p^2L^2}\right)^{\frac{1}{2}} - pL\right]}$$

Now $\left(1 + \frac{R^2}{p^2L^2}\right)^{\frac{1}{2}}$ is of the form $(1 + x)^n$, and if R^2 is small in comparison with p^2L^2, then $(1 + x)^n = 1 + nx$ approximately. The expression in that case becomes

$$\sqrt{\frac{1}{2}pK\left(pL\left(1 + \frac{R^2}{2p^2L^2}\right) - pL\right)}$$

$$= \sqrt{\frac{1}{2}pK\left[\left(pL + \frac{pLR^2}{2p^2L^2} + \right) - pL\right]}$$

$$= \sqrt{\frac{1}{2}pK\left(\frac{R^2}{2pL}\right)} = \sqrt{\frac{pKR^2}{4pL}} = \frac{R}{2}\sqrt{\frac{K}{L}} = \beta$$

As the frequency term p does not come into the last expression, it is said to represent a distortionless condition.

APPENDIX No. 3.

To show that if LS = KR then β = √RS (and that this is the minimum value of β).

In any telephone circuit whatever, we have

$$\beta = \sqrt{\frac{1}{2}\sqrt{(R^2+p^2L^2)(S^2+p^2K^2)}+\frac{1}{2}(RS-p^2KL)} \quad \ldots\ldots\ldots\ldots (1)$$

$$= \sqrt{\frac{1}{2}\times pKR\left[\sqrt{\left(1+\frac{p^2L^2}{R^2}\right)\left(1+\frac{S^2}{p^2K^2}\right)}+\frac{S}{pK}-\frac{pL}{R}\right]}$$

$$= \sqrt{\frac{1}{2}\times pKR\left[\sqrt{\left(1+\frac{p^2L^2}{R^2}+\frac{S^2}{p^2K^2}+\frac{L^2S^2}{K^2R^2}\right)}+\frac{S}{pK}-\frac{pL}{R}\right]} \quad \ldots (2)$$

Now if LS = KR, then $\frac{L^2S^2}{K^2R^2} = 1$, and also $\frac{LS}{KR} = 1$.

Substitute $\frac{LS}{KR}$ for 1 and for $\frac{L^2S^2}{K^2R^2}$ in (2).

This then becomes

$$\sqrt{\frac{1}{2}\times pKR\left[\sqrt{\left(\frac{p^2L^2}{R^2}+2\frac{LS}{KR}+\frac{S^2}{p^2K^2}\right)}+\frac{S}{pK}-\frac{pL}{R}\right]} \quad \ldots\ldots (3)$$

But the quantity $\frac{p^2L^2}{R^2}+2\frac{LS}{KR}+\frac{S^2}{p^2K^2} = \left(\frac{pL}{R}+\frac{S}{pK}\right)^2$

Therefore (3) becomes

$$\sqrt{\frac{1}{2}\times pKR\left[\frac{pL}{R}+\frac{S}{pK}+\frac{S}{pK}-\frac{pL}{R}\right]}$$

$$= \sqrt{\frac{1}{2}\times pKR\times 2\frac{S}{pK}} = \sqrt{\frac{2pKRS}{2pK}} = \sqrt{RS} \quad \text{Q.E.D.}$$

Again (2) is the equation for β in any case, and (3) gives $\beta = \sqrt{RS}$. The only difference between the two formulæ is that in (3) $1 + \frac{L^2S^2}{K^2R^2}$ is replaced by $2 \times \frac{LS}{KR}$ under the inner root sign, and it will be seen by inspection that if $\frac{2LS}{KR}$ is less than $1 + \frac{L^2S^2}{K^2R^2}$ then (3) is less than (2).

This can be proved for

$$\frac{1 + \frac{L^2S^2}{K^2R^2}}{\frac{2LS}{KR}} = \frac{KR}{2LS} + \frac{LS}{2KR} = \frac{1}{2}\left(\frac{KR}{LS} + \frac{LS}{KR}\right)$$

The quantity $\frac{KR}{LS} + \frac{LS}{KR}$ consists of a fraction and its reciprocal, and this combination must be greater than 2 (as will be seen by trial) unless $LS = KR$. In every other case $\frac{1}{2}\left(\frac{KR}{LS} + \frac{LS}{KR}\right)$ is greater than 1, and $\therefore$ $1 + \frac{L^2S^2}{K^2R^2}$ is greater than $2 \frac{LS}{KR}$ (because the former divided by the latter is greater than 1) and (3) is less than (2), and therefore gives the minimum β.

APPENDIX No. 4.

To shew that when $\beta = \sqrt{RS}$ then $\alpha = p\sqrt{KL}$.

As shewn in Appendix 1, formula 8.

$$\alpha = \sqrt{\frac{1}{2}\sqrt{(R^2+p^2L^2)(S^2+p^2K^2)} - \frac{1}{2}(RS-p^2KL)} \quad \ldots\ldots (1)$$

This may be written

$$\alpha = \sqrt{\frac{1}{2}\sqrt{(R^2+p^2L^2)(S^2+p^2K^2)} + \frac{1}{2}(RS-p^2KL) - (RS-p^2KL)} \quad \ldots (2)$$

That is $\alpha = \sqrt{\beta^2 - (RS - p^2KL)}$ ....(3)

Compare with formula (7), Appendix 1.

But when LS = KR, then $\beta = \sqrt{RS}$, proved in Appendix 3.

Therefore (3), above, may be written

$\alpha = \sqrt{RS - RS + p^2KL}$ when LS = KR

that is $\alpha = p\sqrt{KL}$

This simplified formula holds good in any case where pL is great in comparison with R and pK great in comparison with S, as is usual in loaded circuits.

NOTES ON THE DISCUSSION.

MR. COHEN :—

I should like to congratulate Mr. Hill on this admirable paper, and I am sure we are all indebted to him for the clear and complete manner in which he has put the information on aerial loading before us.

It is satisfactory to see that aerial loading shews signs of proving itself in, in this country, provided that the line is sufficiently long.

A point of considerable importance which it would appear still requires settlement, is the definite determination of the value S/K, and therefore the value of equated length in M.S.C., which should be taken as a working minimum in connection with transmission standardisation of aerial loaded lines when working as part of the trunk network of the United Kingdom. We are, of course, always forced to take reasonable minimum values when determining the lay-out of telephone plant on the basis of its talking efficiency, and it is of great importance from the standpoint of expenditure in the case of aerial loading that the transmission standardisation is not based on a maximum value of S/K, which only occurs on, say, two or three days in the year. On the other hand, it is equally necessary to ensure that the overall transmission is only worse than the standard laid down for the minimum time consistent with reasonable expenditure.

It would, therefore, appear that a comprehensive series of A.C. leakance tests spread over a complete year are required in order to assist in determining this.

If there is anything in this paper that I should feel inclined to criticize, it is that the effects of attenuation and reflection are, perhaps, not as sufficiently separated as they might well be (and, in some cases, it is rendered more difficult to effect this separation owing to the omission of S/K values applying when the particular tests were carried out). Table 5 is of particular interest, and in this case I have been able to get from Mr. Hill the S/K value applying.

In the P.O.E.E. Journal for April last year, I gave some reflection loss curves for loaded lines with values ranging from 800ω up to 3000ω for the real portion of the impedance. In the same paper is a curve for the losses occurring at the junctions of two loaded lines of different impedances, which is termed transition loss.

Now these curves were obtained from a large number of tests of loaded lines, both actual working lines and laboratory cases, and although all taken on cases of cable loading, there does not appear to be any particular reason against their application to the general loading problem.

I have worked out the losses from these curves for the use of O.W. loops, C.B. terminals, in table 5 of Mr. Hill's paper, and the following are the results:—

		M.S.C.
Equivalent of No. 6 ...	...	5·50
do. 7 ...	...	6·25
Transition loss	...	0·30
Terminal loss, No. 6 ...	...	4·20
do. 7 ...	...	3·30
Total	...	19·55

to compare with 21·4, which is fair agreement, considering that the distribution of the leakance plays an important part and also that no two C.B. terminals used are likely to have exactly the same impedance values. It is unfortunate that the other tests appear to have been made with ohmic Resistance loops, as these give no indication of the true tapering effects of an actual subscriber's cable loop.

This paper demonstrates the well known difficulty of carrying out accurate tests on long trunk lines, for example, we see that a series of tests taken during the winter months had to be cancelled owing to faults.

Then again, it is pointed out that the distribution of leakance is very rarely uniform over a long aerial line and this leads to errors.

A length of 200 miles of artificial aerial line, with 400 lb. and 200 lb. equivalent conductors, and with variable insulation resistance from infinity down to 100,000ω per mile, has been constructed by the Mount Pleasant Factory for the Research Section. It is hoped that this artificial line will prove of use in throwing light on a number of problems in connection with open wire lines.

MR. NOBLE:—

I had no intention of speaking this evening for the reason that I saw the Paper was a rather lengthy one and there would be very little time left for discussion. I thought, therefore, Mr. Cohen, who had expert knowledge, should occupy the time.

As you are aware, Mr. Chairman, all the Sections in the Headquarters are so fully loaded that they have not all time to study the work of another particular Section, and they are, therefore, grateful to any gentleman who gives them information in tabloid form, and I am sure they will be very grateful to Mr. Hill for the excellent contribution which he has given us this evening.

Like Mr. Cohen, I think it would be well if such papers as Mr. Hill's, which are highly technical and important, were printed in advance. I am sure the Institution would be only too pleased to do so.

There are many papers which the authors have to prepare at such short notice that there is no time to have them printed, but as Mr. Hill remarked towards the close of his Paper, when he started with these experiments in Mr. Kempe's time, I should think that with regard to this very Paper it might have been printed in advance and circulated so that the discussion would have been of more value.

I have only one other point to mention in conclusion and that is, the Institution should be very pleased to have gentlemen like Messrs. Hill, Martin, Pollock and Hay, and others, who are keeping the British Post Office so advanced on this very important subject of loading of underground and open lines.

I hope that either Mr. Pollock or Mr. Hill will contribute a Paper to the Institution on the loading of the Leeds-Hull cable, which has just been completed.

MR. HART:—

I should like to associate myself with the previous speakers in congratulating Mr. Hill on his admirable Paper, and should like to say that I think our thanks are especially due to Mr. Hill for giving us another Paper which will be a standard work of reference for us. I think the Paper, more than anything, shows how much the success of transmission work, particularly on aerial lines, depends upon the practical men, who are responsible for the maintenance of such lines, thoroughly understanding the conditions. Taking, for instance, the value of the leakance factor which has been mentioned; cracked insulators, defective potheads, dusty and dirty main frames, all stand out as factors in the determination of the leakance.

I rather hoped that Mr. Hill would have given us a few notes on the aerial lines in Germany as, I believe, he has actually seen some of these lines. I have seen various illustrations in technical papers on Loading in Germany, which seem to indicate that loading coils are built up in the insulators. There is an absence in that case of the rather cumbrous looking cases which we have seen in this connection. There is also an absence of covered leads connecting the main conductors with the coils. My experience of aerial line construction and maintenance has somehow or other bred in me a distrust for covered leads on poles. I have the feeling that directly the line leaves the insulator the certainty of maximum efficiency disappears. Consideration will probably be given to the use of the self-contained insulator form of loading coil, but now that the London-Leeds experiments have brought aerial loading out of the theoretical to the practical, the results are, I think, very encouraging.

The weather variations appear to have caused variations in leakance as measured by direct current, but the variations as measured by alternating current did not correspond. The direct current variations are, of course, due to variations at the insulators. Perhaps Mr. Hill will tell us whether we may assume that the alternating current measurements of the leakance are in some degree independent of the condition of the insulators, *i.e.*, we have to consider, even in the case of an aerial line, the dielectric.

Mr. Hill said the selected circuits are free from underground, but presumably he has not overlooked the fact that there were some leading-in points. My only object in raising this point was to ask Mr. Hill what measure of importance he attaches to the leading-in points on aerial lines, leaving out the consideration of the localising facilities, and dealing only with their effect on the general efficiency of the circuit.

Now as regards the lines suitable for loading. The longest direct aerial circuit we have in England is 450 miles, and the longest aerial circuit that we ever make up in every-day use is about 700 miles. I take it that if these circuits were loaded we should be able then to get the maximum benefit. If we look at the Standard Cable Equivalent for such a circuit (*i.e.*, the theoretical S.C.E. required for a 700 mile circuit), we cannot allow any more than 20 miles of Standard Cable to get commercial speech. To get that efficiency, we have to erect 600-lbs. conductors. Perhaps Mr. Hill would indicate what could be done or what might be looked forward to in the future, with a view to obviating the necessity for erecting very heavy conductors. We have got for the 700 mile length which I have in mind 600-lbs. conductors which are joined in series, but it is impossible to make up a circuit continuously of 300-lbs. Perhaps there are some 200-lbs.

In connection with the comparative costs of loading coils, Mr. Hill mentions the economy of the cheaper ones. We have often to decide against loading comparatively short lengths of underground because the cost of the loading is greater than the cost of the additional copper, and up to the present time we have always followed the maxim (I am not sure whether it is due to Lord Kelvin or Mr. Heaviside), which practically amounts to this: "When in doubt, lay down the copper." It seems that if the cheaper loading coils could be brought in that we shall be able to alter that maxim.

MR. LEE:—

I shall be glad if Mr. Hill will kindly give us his views, based upon his wide experience of speech tests on cables, as to whether phase distortion causes distortion of speech. That is, I am referring to the well known fact that a circuit which is not "distortionless" according to the mathematical definition, the waves of different frequencies travel at different rates along the circuit, hence the harmonics become displaced with reference to the fundamental. It is, I think, agreed by practical experiments, that amplitude distortion is the more serious, that is, the effect caused by waves of various frequencies being attenuated differently. Several well known physicists apparently differ in their views of the question of phase distortion, and it would be interesting to know whether the problem can be solved by means of telephone tests.

I had the pleasure of being present at some of the tests described by Mr. Hill in his paper, and was very much impressed by one apparent difficulty which occurred by making the comparison speech tests of loaded aerial lines against standard cable. This was that the "tone" or "pitch" of the speech received over standard cable differed markedly from that received on the aerial line. From the practical point of view the ease with which the speech is understood is a measure of the efficiency and the standard cable test measures this quantity fairly well, but at the same time the difference in "tone" is so marked that it appears to be desirable to adopt the more scientific methods of comparison which are available, whenever possible.

The insulation tests of the lines used by Mr. Hill for the loading experiments appears to be fairly good, but it would be of interest to know whether these lines were specially overhauled beforehand. It is quite evident from the results of the tests that the future of aerial loading in this country is dependent upon whether a high average standard of insulation can be maintained.

Perhaps Mr. Hill would also supplement the information in regard to the alternating current constants of the lines under test, by giving the direct current constants of these lines also.

A further point in connection with the alternating current tests is whether the magnitude of the testing current had any effect on the inductance of the loading coils under test, and perhaps Mr. Hill will kindly state this. It will be borne in mind that the method of test employed provides a standing or stationary wave on the circuit with nodes and antinodes of current, and there may be points of the circuit at which the current exceeds that sent out by the alternator

The loading question, in this country, at present suffers from the disadvantage that the apparatus used on telephone circuits has been designed for use with non-loaded circuits, and it may be found desirable in the future to increase the impedance of telephone apparatus, at the same time making it more efficient, in order to reduce the reflection effects.

MR. ALDRIDGE :—

While agreeing with the speakers as to the value of this Paper, I think it would be enormously increased if Mr. Hill were to give us details showing the results of the tests day by day.

Taking the figures as he has given them, there is still quite a considerable discrepancy between the values of the attenuation constant and the values of S/K on page 37. I notice that the attenuation is sometimes greater as the leakance is less. Mr. Hill states that this may be due to the irregular distribution of the leakance, and I think this is most probably the reason, and all tests, I think, on long lines should be made section by section so that an error can be allowed for. I might mention that taking No. 6 Trunk with a uniform change, the equated length comes out at about 5 standard miles; assuming all that leakance concentrated in the centre, the allowance is only about 4—quite a large percentage improvement.

There is one other point in connection with this test. I believe it makes a lot of difference to the values depending on the position of the test point relative to the first loading coil. I should like to know Mr. Hill's views on that. Possibly there is a lot of difference between figures you get from the place of the testing point depending on how far away the first loading coil is.

Mr. Hill, on page 37, refers to special tests. I believe these tests were made on speaking a long "Ah!" into his transmitter—one cannot call that speech, and I should like to ask if Mr. Hill has any tests showing the equivalent of it. From tests made at Telephone House, it seems that it is not the same thing and one would imagine this method would result in too high frequency, and this is borne out to some extent by the tests on page 37. The losses there are greater than one would expect from speech, though corresponding to the higher frequency.

Then Mr. Hill, on page 41, has given a formula for getting the attenuation of a short cable. That formula gives the apparent attenuation for a short length of cable terminated by two impedances. Well now, this is not the same case, I think, as a piece of cable or open line terminated by two cables. The cable at the sending end does not act in the same way as that at the receiving end. I have taken the case he has given for No. 6 trunk, as it is the only one I have any figures for, and I get a loss of 2·8. Mr. Hill's outside figure was 3·8, and the formula he gives gave 1·4 I think future advances in this subject are to be found by testing each piece of apparatus in each section of line bit by bit, and only in that way are we likely to get results according with theory. If we do not, we cannot predict accurately the action of the circuit.

MR. HAY:—

The equivalent frequency of speech is usually taken as 800 periods per second, and Mr. Hill has apparently adopted this value. I have been under the impression for a considerable time that the discrepancies which exist between observed and calculated values in connection with cable measurements indicate that, for one thing, the equivalent frequency taken for speech is too high. On making some preliminary measurements of the equivalent frequency of the "standard voice" by methods which are direct and which I shall describe presently, my impressions have been confirmed. I found that in exceptional cases only does the equivalent frequency of speech approach 800 periods per second, and is in general much lower. Further, it varies with the character of the circuit in which the voice currents are flowing, which is only what would be expected seeing the changes which occur to the transmitted pulses as they progress through the circuit. It would be most remarkable if anything like a constant value for the equivalent frequency of speech could be obtained in all circumstances. It is of fundamental importance that the equivalent frequency of speech should be determined as accurately as possible for all conditions, as otherwise results obtained using a doubtful value are vitiated.

The expression equivalent frequency of speech used herein is defined as that particular frequency of alternating voltage, which, when applied to a transmission circuit either by means of the standard voice or of a sine-wave alternator, gives exactly equal electrical results. The standard voice is the voice which is usually used in comparing telephone receiving transmitters, lines, etc.

The experiments undertaken in order to determine the equivalent frequency defined as above were carried out as follows:—

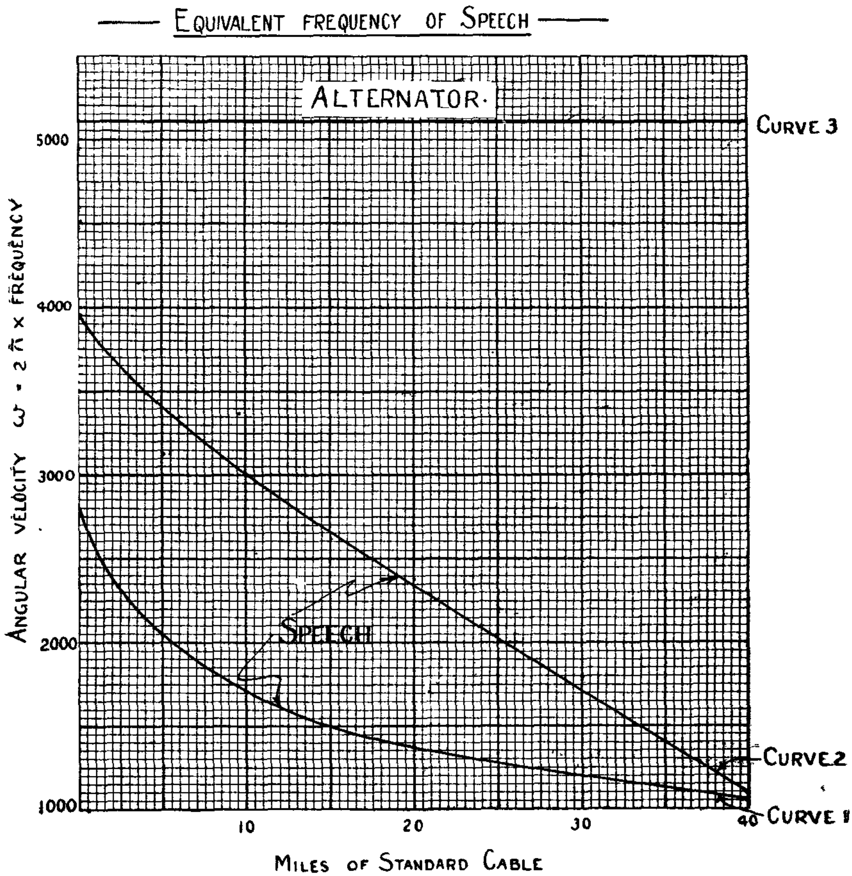
(1) A good mica condenser of 1 microfarad was introduced into the transmission line, whose length was varied. The voltage across the condenser and the current in it was measured by means of thermo instruments, and the frequency obtained by means of the relation

$$\frac{C}{V} = K\omega$$

The results are shown by curve 1.

(2) A loading coil was then substituted for the condenser and a similar set of observations made, from which the frequency was obtained by means of the relation

$$\frac{V}{C} = L\omega$$



the resistance of the loading coil, being so small as to be negligible in these circumstances. The results are given by curve 2.

In all cases the frequency of the alternator, which was arranged to run at $\omega = 2\pi \times 800$, was taken for all conditions and the results as expected appear as a horizontal line (curve 3).

It was a most unfortunate accident which caused investigators to use the symbol β for the attenuation constant, and it is hoped that its use will not be perpetuated by Mr. Hill. In his effort to preserve its use he has broken through mathematical convention and makes himself responsible for the unsymmetrical expression $(\beta + j\alpha)$. Long usage will never justify writing a complex quantity in this manner.

MR. G. M. SHEPHERD :—

[COMMUNICATED.]

I have a few remarks to make in regard to your very interesting paper on loading that, perhaps, can best be communicated.

There is very little destructive criticism to make. I think that in dealing with the distortionless line $\frac{L}{R} = \frac{C}{S}$ which figures prominently in your paper, you should have made reference to Heaviside, who is the author of almost all that is known of this peculiar condition. It is, in fact, scarcely too much to say that there is hardly anything concerning the theory of transmission that cannot be found somewhere in Heaviside's work ; though it may need some excavating to reach it. Now concerning terminal reflection, I rather gather from your remarks that the state of our knowledge on this subject is not as definite as it should be, at least as regards aerial loading, and I venture to suggest that as we now possess a 200 mile artificial line, an exhaustive laboratory investigation should be undertaken to ascertain the losses, and construct working curves to correspond with those already existing for cables.

I cannot quite follow the formula cited on page 41, but know that the following expression is more or less valid for the transition loss between two line sections equivalent to not less than, say, 7 m.s.c.,

$$b = \log_e \frac{1}{2r}\sqrt{1 + r^2 + 2r\cos\theta}$$

$r/\underline{\theta}$ being the impedance ratio, and "*b*" an attenuation to be added to that of the lines. I communicated this formula to the discussion on Mr. Aldridge's paper.

The question of the minimum length worth loading is most important, and though you direct attention to it, I have never seen any attempt to exactly specify the maximum load for any given conditions. One way of doing this is to express the β of the line, and the terminal loss, as functions of L, the added inductance, and then find the minimum equivalent. Thus one might write—

$$\text{m.s.c.} = \frac{l}{a_1} f(\text{L}) + \text{P}(\text{L})$$

where a_1 is the standard cable β at 800 ∿, and l = length of line loaded.

Considering now an aerial line of l miles, $\frac{\text{S}}{\text{C}} = 250$, having terminals of 3 miles of 20-lbs. cable loop. Assuming *u.g.* cable coils to be used, the losses dependent on the added L can be written

$$\text{R}_1 = 300\,\text{L} + \cdot 9;$$

and the terminal loss function is thought to be something like this—

$$\text{F}(\text{L}) = 25\sqrt{\text{L} + \cdot 0037}$$

whence

$$\text{m.s.c.} = 9{\cdot}45 l \left\{\left(\frac{\text{R}}{2} + 150\text{L} + \cdot 45\right)\left(\frac{\text{C}}{\text{L}} + \cdot 0037\right)^{\frac{1}{2}}\right\} + 25\left(\text{L} + \cdot 0037\right)^{\frac{1}{2}}$$

and the minimum is given by—

$$\text{L} = \frac{l\sqrt{c}\,(4{\cdot}725\text{R} - 6{\cdot}24) - \cdot 0925}{1415\, l\sqrt{c} + 25}$$

which enables one to see clearly the necessity for either abolishing the terminal loss, or only loading lines of considerable length. Such a formula is, of course, quite empirical, but would be very useful if we were entirely certain of the shape of the loss curves.

I am glad you suggest the use of the small cable coils for aerial work. I have often thought that the large

#501 pattern would be a costly luxury for the conditions found in this country.

MR. HILL'S reply:—

I thank the various speakers for their generous appreciation of my efforts in connection with this Paper.

The following is my reply to the various questions and criticisms on it:—

Mr. Cohen points out that $\frac{S}{K}$ is not always given. Well, generally it is given, but I find there are some cases where it could not be obtained, owing to the wires having to be given up for commercial service before the tests were completed. I was, however, able to furnish additional information in one case to Mr. Cohen, as he has pointed out. As a rule sufficient data are given to enable the reflection losses to be arrived at.

It is interesting to note that Mr. Cohen finds that the terminal loss I gave for the open lines, with C.B. apparatus at the ends, agrees within permissible limits with results made up from curves given by him in the P.O.E.E. Journal. I notice from the P.O.E.E. Journal, quoted by Mr. Cohen (April, 1913, page 17), that the results were deduced from tests on lines from 3 to 17 S.M. I should not expect lines of so small a value as 3 S.M. to give uniform losses comparable with those on lengths of 17 S.M., and this has raised the doubt in my mind, as to whether this circumstance might have in some degree caused the discrepancy.

I may say that in the case of the artificial standard cables used for terminal effects, the resistance and capacity were uniformly disturbed along the whole cable. They were not mere resistances.

I agree with Mr. Cohen that the artificial line he mentions would be a valuable adjunct in tests of long distance aerial lines.

In reply to Mr. Noble. It seems from Mr. Noble's remarks that our work of experiment and research is closely followed and fully appreciated by our chiefs. Although I did not doubt that, I am confident that this assurance will act as an incentive to those engaged in such work, to make unceasing efforts in the path of improvement.

Mr. Hart asked for information as to German methods of aerial loading by the use of small coils. I have not, personally, seen these lines, but I have had three slides prepared from an Article by Dr. Ebeling, of Messrs. Siemens and Halske, in 1910. Possibly this is what Mr. Hart refers to. It is obvious from these slides that the German coils are much smaller than the English ones. It may be deduced, however, from the data given in Dr. Ebeling's Paper that the German coils have a resistance of 79ω per henry at 900 periods per second, as against 25ω per henry at 800 periods in the case of the coils used in our experiments. In these circumstances, no useful comparison is possible. It is certain that much smaller coils than the ones we used could be made by the contractors if they were allowed the same resistance, but this would be inadmissible, and I believe that much more efficient coils than those shewn in the slides, are now produced in Germany, but they may be expected to be proportionately larger. It may be mentioned that the coils we use here are made to withstand 8000 v., and also that the wire windings are stranded and insulated. These features involve some increase in size. The coil we are using is among the most efficient in the world, but I have the distinct impression the German ones are efficient for their size. I may say that the V.I R. leads, used in connection with our coils, have maintained a very high insulation up to the present. The Western Electric Co. inform me that this type of wire gives satisfaction in America.

As to the difference between the insulation measured with direct current, and at A C frequencies, I should prefer to extend the experiments before giving an opniion. In order to obtain an aerial loaded circuit equal to a 600-lbs. circuit unloaded in English Winter conditions, it would be necessary to load a 400-lbs. circuit, assuming $\frac{S}{K} = 250$. Even then the reflection losses at the end would require to be added. A 700-miles 400-lbs. loaded open wire, allowing for end losses with terminal transformers but not intermediate underground or leadings-in, would approximate to 20 M.S.C.

I am of opinion that leading-in at intermediate offices should be kept down to a minimum. The losses introduced by such arrangements are often much greater than their Standard Cable equivalent would imply, owing to the introduction of inductive disturbances caused by low insulation, bad joints, etc. As a matter of fact, there are a number of leading-in points on the loaded aerial line. I mentioned in my Paper that a number of tests had to be cancelled through a fault. To locate this fault after other means had failed, all the leading-in points were cut out. After their restoration the fault had disappeared, but on enquiry we learned that all had been found normal and nothing altered. This was, to say the least, very surprising, and I think it shows the danger of leading-in.

As regards the use of cheaper loading coils. I think there is a case for study as regards cable circuits 40-lbs. and under in weight per mile. If a coil could be produced having $\frac{R_1}{L_1}$ equal say to 100, at half the cost of the present underground coil, and taking up half the space of the present standard underground type, possibly good use could be found for it. I had already made some enquiries in this direction, but although the prospect seems favourable at first sight, further investigation is necessary before a final reply can be given.

In reply to Mr. Lee, I have no specific information as to whether phase distortion causes distortion of speech. Some well known scientists think it does not. I should expect, however, that if the distortion exceeds certain limits, it must cause confusion of such sounds as rapidly follow each other. I have not, however, found it possible in speech tests to separate amplitude distortion from phase distortion. I agree that the most scientific methods should be used, and I think I may say this was, as far as possible, done is this case. Practical tests should, however, also be included.

Mr. Lee asks if the circuits used for the experiments were specially overhauled beforehand. They were not, but they were carefully tested for insulation, and found to be in good order, as verified by the subsequent tests dealt with in the paper.

He also asks for the constants of the circuits measured by direct current. R and S are of course given, and I supplemented the information as to the capacity when reading my Paper, by stating that it did not materially differ at direct and A.C. measurements. The inductance, however, still remains to be accounted for. An attempt was made to measure it, but a delicate mirror galvanometer had to be used, and it was found impossible to get any steady readings owing to interference from induced currents.

A further question by Mr. Lee is—Can the effect of the variation of the testing current on the inductance of the coils be stated? The change of inductance of these coils with current is very small, and as the testing current was generally kept down under one mlmp, only a small change was expected. Some special tests made by varying the amount of the testing current failed to reveal any change in the efficiency of the circuit. As a matter of fact, calculation in this case shews that if the inductance varied as much as 50% the β would only vary 6%.

When loading becomes more general, I agree with Mr. Lee that it may be found advantageous to increase the impedance of Telephone Subscribers' Sets.

Mr. Aldridge thinks that tests on long lines should be made section by section, and that by such means practice and theory would more nearly coincide. There are, however, difficulties in the way in this case, where the loading points are 8 and 12 miles apart respectively. It is found difficult in such circumstances to find a suitable testing point where the loading would be normal, *i.e.*, the coils are generally either too near or too far away. If only short lengths are tested, the resulting error due to abnormal end lengths is proportionately greater on short than on long lengths.

A second point taken up is that Mr. Aldridge believes that the position of the first loading coil makes a considerable difference to the test. I think it must make some difference, especially if the length of line is very short. The recorded results, however, of speaking tests from Harby where normal spacing obtains, and St. Albans where it does not, shew approximately the same result with 10 M.S.C. as terminals.

Next, as to the use of a prolonged "Ah!" instead of counting in speaking tests. Mr. Aldridge wishes to know what is the equivalent frequency of the "Ah!" as he thinks it too high. Well, we have definite tests on record which sufficiently answer his query, particularly as regards testing of loaded lines. The most recent is the new Irish loaded cable. This cable was measured by A.C. at 800 periods per second. Based on the βl measurement the Standard Cable value is 9·0 S.M., and this was found to be the case by a speech best when using "Ah!" Making all allowances for reflection the agreement is good. Again, in the tests on the open lines, in this Paper, the results of speech tests on the unloaded aerial lines are fairly good. It might be mentioned that both methods ("Ah," and counting) were tried. It must not be overlooked that what was attempted was a comparison of volume, and that counting introduces articulation. Finally, in view of the fact that the test is merely comparative, any volume error would affect both the standard cable and the line, and it is not clear why any considerable error should be looked for in comparing the loaded aerial line with the standard cable, as there is considerable distortion in both.

Mr. Aldridge thinks that the formula I use on page 41 does not apply as I have used it, but that it gives the apparent attenuation for a short length of cable terminated by two impedances, and he thinks it is not the same case as a piece of cable or open line terminated by two cables. Now I totally disagree with Mr. Aldridge. Instead of being terminated by two impedances, I have in this case supposed it to be terminated by two long cables, just the opposite of Mr. Aldridge's contention. The formula I gave, moreover, applies to any inserted length whatever. If that length is sufficiently long

long line (Impedance Z_0)	×	long line Impedance Zr	×	long line Impedance Z_0

we have a case for which Mr. Aldridge has himself calculated some reflection curves. I find that the formula on page 41 reproduces the results indicated by Mr. Aldridge's curves. If, however, his criticism of the formula I have used is correct, this could not be unless his curves are wrong. I conclude that his criticism is wrong.

[Mr. Aldridge has since informed me that he misinterpreted this formula and withdraws his remarks on it.]

It will be convenient if I deal with Mr. Shepherd's communication respecting the same formula at this stage. That gentleman writes—"I cannot quite follow the formula cited on page 41, but know that the following expression is more or less valid for the transition loss between two line sections equivalent to not less than, say, 7 M.S.C.

$$b = \log_e \frac{1}{2r} \sqrt{1 + r^2 + 2r\cos\theta}$$

$r/\underline{\theta}$ being the impedance ratio and "b" an attenuation to be added to that of the lines." It can easily be shown that th's formula must give similar results to that on page 41, for long lines, and in the large number of cases where the angle is small (loaded l nes and open lines) it is identical. To prove this, let $\cos\theta = 1$ (*i.e.*, let the angle θ be negligible), then the formula becomes

$$b = \log_e \tfrac{1}{2} r \sqrt{1 + r^2 + 2r \times 1}$$

$$= \log_e \tfrac{1}{2} r \sqrt{(1 + r)^2} = \log_e \frac{1 + r}{2r}$$

$$= \log_e \frac{1 + \frac{1}{r}}{2}$$

This gives the reflection at one point. To make it comparable with the formula on page 41, it must be modified for application at a second point when it becomes $\frac{1 + r}{2}$ and the sum of the attenuation at those points is

$$\log_e \frac{1 + \frac{1}{r}}{2} + \log_e \frac{1 + r}{2}$$

Now $\frac{1 + r}{2} + \frac{1 + \frac{1}{r}}{2} = \frac{2 + r + \frac{1}{r}}{2}$

But r is the ratio of the Impedance, *i.e.*, it is $\frac{Z_0}{Zr}$, therefore the formula becomes

$$\frac{2 + \frac{Z_0}{Zr} + \frac{Z_0}{Zr}}{2} = 1 + \frac{1}{2}\left(\frac{Z_0}{Zr} + \frac{Z_0}{Zr}\right)$$

$$= 2\left[\frac{1}{2} + \frac{1}{4}\left(\frac{Z_0}{Zr} + \frac{Zr}{Z_0}\right)\right]$$

Now the formula in the brackets is that part of the formula on page 41 of my Paper, which applies to long lines. To complete the proof of identity, call $\frac{1+r}{2} = A = fr$

$$\frac{1+\frac{1}{r}}{2} = B = f_1 r, \text{ then } 1 + \frac{1}{2}\left(\frac{Z_0}{Zr} + \frac{Z_0}{Zr}\right) = A + B.$$

It can be shown that

$$\log_e A + \log_e B = \log_e \frac{A+B}{2}$$

that is $\log_e \frac{1+r}{2} + \log_e \frac{1+\frac{1}{r}}{2} = \log_e \left[\frac{1}{2} + \frac{1}{4}\left(\frac{Z_0}{Zr} + \frac{Zr}{Z_0}\right)\right]$

where $r = \frac{Zr}{Z_0}$. Q.E.D.

I cannot, however, follow Mr. Shepherd's treatment of the angle θ.

Mr. Hay has made experiments to determine the equivalent frequency of human speech, and he finds it to be much less than 800 periods per second. He says the expression "epuivalent frequency of speech is defined as that particular frequency of alternating voltage which, when applied to a transmission circuit by means of the Standard Voice, or a sine-wave alternator, gives exactly equal electrical results."

Mr. Hay measured the Impedance of a condenser and an Inductance in a line which was being spoken into, and from that he determined the frequency. In practice, speech is, of course, received through a terminal telephone, and the observed attenuation is based on observations which involve electrical, magnetic mechanical, acoustic, and physiological effects. I think this is quite a different case to that dealt with by Mr. Hay. As to why we should get a difference in the two cases is a matter for investigation, but one cannot, in my opinion, argue from the one to the other. For example, the ear may be more sensitive to the upper than the lower notes and the elimination of upper notes which takes place may give the effect of increased attenuation, also the diaphragm will tend to resonance at some frequency The concensus of opinion of competent authorities is that a frequency of 800 periods represents the attenuation due to speech for practical purposes. I think, however, that one should maintain an open mind on this subject in the present state of our knowledge.

Mr. Hay thinks it unfortunate that I perpetuate the use of β as an attenuation constant. He prefers α as more logical. It is well known that β was formerly in general use as the attenuation constant, in England and elsewhere, and that it is still used by a considerable majority of writers throughout Europe. I think Pupin was probably the first to use it, in America. If we take a sufficiently broad view of the matter, we must admit that the use of α in such circumstances would only lead to confusion. Until the matter is decided definitely by an international conference, I think no alteration should be made, *i.e.*, β should be used.

Mr. Shepherd is of opinion that Heaviside should be referred to in connection with the distortionless condition, as he is the author of all that is known of this particular condition. I fully agree as to Heaviside's wonderful work, but I thought his achievements were so well known that to emphasise them would be like gilding fine gold or somewhat like acknowledging Ohm in connection with Ohm's law. Still, I am pleased to place our common obligation to him once more on record. Mr. Shepherd suggests an exhaustive test on a new artificial cable representing an aerial line which is now available. Mr. Cohen referred to this. I think it would be a very useful adjunct if its equivalence to aerial lines is verified by comparison with them, more particularly the variation of the electrical constants with frequency.